Error-bounded and adaptive reconstruction

Reconstruction is the process of recovering a continuous signal from a set of samples. Continuous signals are very common; some examples are radio waves and microwaves (one-dimensional signals), photographs and medical images (two-dimensional signals), and scenes generated by computer graphics or medical imaging (three-dimensional signals). Whenever a continuous signal is processed by a computer, it must be sampled so the computer can store it. During digital signal processing the continuous signal must be frequently reconstructed. This is necessary whenever the sampled signal is subjected to a resampling operation, such as rotating or rescaling an image, mapping a texture onto a polygon, or rendering an image from a medical dataset.

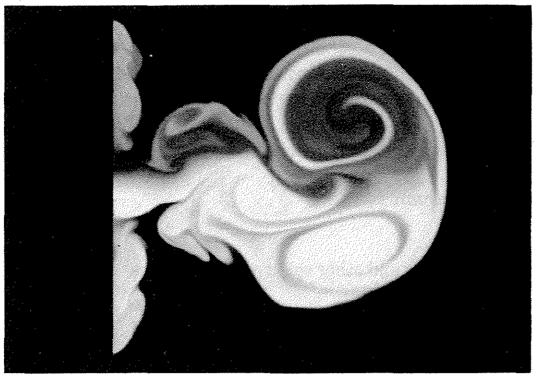
According to Shannon's sampling theorem, if the reconstruction kernel is a sinc function, and if the samples are taken at or above twice the rate of the highest frequency in the signal, then the reconstruction will be perfect: the reconstructed continuous signal will exactly match the original continuous signal. However, in practice, sinc functions are not usually used as reconstruction kernels because they have an infinite width. A variety of other reconstruction kernels have been used, such as box-, tent-, cubic-spline-, hyperbolic-, and truncated- or windowed-sinc kernels. Each kernel contributes a certain amount of error to the reconstructed signal.

We have developed a reconstruction technique in which we measure the amount of this error. We use a Hamming window—a suitably windowed truncated sinc function—as a reconstruction kernel. By varying the width of the kernel, we vary the amount of reconstruction error at each discrete sample point. When we use these error measurements to alter the filter size, we reconstruct a signal within a certain error bound: the signal is guaranteed to have a reconstruction error below the bound, at every discrete sample point. In the limiting case where the error bound is set to zero, the sinc kernel has an infinite extent.

Our method adapts both to the position of the reconstructed point and to the data near the reconstructed point. The position adaptation adjusts the kernel width according to the distance between the re-sampled point and nearby data points. The larger this distance the wider the kernel needs to be to fall below an error bound. The data adaptation measures how the data itself contributes to the reconstruction error. In regions of large data values a wider kernel is needed, while regions of smaller data values require a narrower kernel to fall below an error bound.

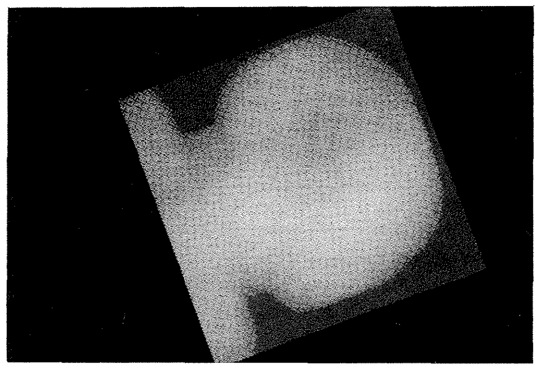
Our method measures reconstruction error in the spatial-domain; in almost all similar work reconstruction error is measured in the frequency domain, and the source data is not considered in the error computation. Thus, with our method it is not necessary to take a Fourier transform of the signal; such transforms can be expensive for multidimensional signals.

We have implemented our reconstruction method and tested it by transforming 2D images. Figure 1a shows an image that, in figure 1b, has been reconstructed onto a grid rotated by 30° and resampled at 75 percent of the original resolution. Figure 1c is an image of the filter sizes used to reconstruct each pixel, where bright values stand for larger filter sizes. Notice both the diamond cross-hatching pattern, which indicates that the filter size is adapting to position, and the outline of the figure, which indicates that the filter size is adapting to the data.



(a)

(b)



(c)

Figure 1. (a) Original image. (b) Image rotated by 30° and scaled by 75 percent. (c) Filter sizes used for the resampling. Bright values stand for larger filter sizes.

For more details, see Raghu Machiraju, Edward Swan, and Roni Yagel, "Error-Bounded and Adaptive Image Reconstruction," Computer And Information Science Research Center Technical Report Number OSU-CISRC-1/95-TR03, *The Ohio State University, January 1995.*

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