Experimental Design and Analysis for Human-Subject Visualization Experiments

IEEE Visualization 2007 Tutorial

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Schedule

8:30 – 10:10 AM	**100 minutes**	**Experimental Design and Analysis Part I**
10:10 – 10:40 AM	**30 minutes**	**Coffee Break**
10:40 – 12:10 AM	**90 minutes**	**Experimental Design and Analysis Part II**

Motivation and Goals

- Course attendee backgrounds?
- Studying experimental design and analysis at Mississippi State University:
  - PSY 3103 Introduction to Psychological Statistics
  - PSY 3314 Experimental Psychology
  - PSY 6103 Psychometrics
  - PSY 8214 Quantitative Methods In Psychology II
  - PSY 8803 Advanced Quantitative Methods
  - IE 6613 Engineering Statistics I
  - IE 6623 Engineering Statistics II
  - ST 8114 Statistical Methods
  - ST 8214 Design & Analysis Of Experiments
  - ST 8853 Advanced Design of Experiments I
  - ST 8863 Advanced Design of Experiments II
- 7 undergrad hours; 30 grad hours; 3 departments!

Motivation and Goals

- What can we accomplish in one morning?
- Study subset of basic techniques
  - I have found these to be the most applicable to visualization evaluation
- Focus on intuition behind basic techniques
- Become familiar with basic concepts and terms
  - Facilitate working with collaborators from psychology, industrial engineering, statistics, etc.

Outline

- *Empiricism*
- Experimental Validity
- Experimental Design
- Gathering Data
- Describing Data
  - Graphing Data
  - Descriptive Statistics
- Inferential Statistics
  - Hypothesis Testing
  - Hypothesis Testing Means
  - Power
  - Analysis of Variance and Factorial Experiments

Why Human Subject (HS) Experiments?

- Graphics hardware / software more mature
- Sophisticated interactive techniques possible

- Focus of field:
  - Implementing technology $\rightarrow$ using technology
  - Trend at IEEE Virtual Reality, SIGGRAPH
  - Called for in *NIH-NSF Visualization Research Challenges Report* [Johnson et al. 06]

- Increasingly running HS experiments:
  - How do humans perceive, manipulate, cognate with CG-mediated information?
  - Measure utility of visualizations for application domains

Conducting Human-Subject Experiments

- **Human subject experiments at IEEE Visualization:**

Year	Vis Papers	%	Info Vis papers	%
2006	8 / 63	13%	2 / 24	8%
2007	3 / 56	4%	12 / 27	44%

- **Human subject experiments at IEEE Virtual Reality:**

VR year	papers	%	sketches	%	posters	%
2003	10 / 29	35%			5 / 14	36%
2004	9 / 26	35%			5 / 23	22%
2005	13 / 29	45%	1 / 8	13%	8 / 15	53%
2006	12 / 27	44%	2 / 10	20%	1 / 10	10%
2007	9 / 26	35%	3 / 15	20%	5 / 18	28%

Logical Deduction vs. Empiricism

- Logical Deduction
  - Analytic solutions in closed form
  - Amenable to proof techniques
  - Much of computer science fits here
    - Computability (what can be calculated?)
    - Complexity theory (how efficient is this algorithm?)
- Empirical Inquiry
  - Answers questions that cannot be proved analytically
  - Much of science falls into this area
  - Antithetical to mathematics, computer science

What is Empiricism?

- The *Empirical Method*
  - Develop a hypothesis, perhaps based on a theory
  - Make the hypothesis testable
  - Develop an empirical experiment
  - Collect and analyze data
  - Accept or refute the hypothesis
  - Relate the results back to the theory
  - If worthy, communicate the results to scientific community

- Statistics:
  - Foundation for empirical work; necessary but not sufficient
  - Often not useful for managing problems of gathering, interpreting, and communicating empirical information.

Where is Empiricism Used?

- **Humans are very non-analytic**

- **Fields that study humans:**
  - Psychology / social sciences
  - Industrial engineering
  - Ergonomics
  - Business / management
  - Medicine

- **Fields that don't study humans:**
  - Agriculture, natural sciences, etc.

- **Computing Sciences:**
  - Human-computer interaction
  - Software engineering

Experimental Validity

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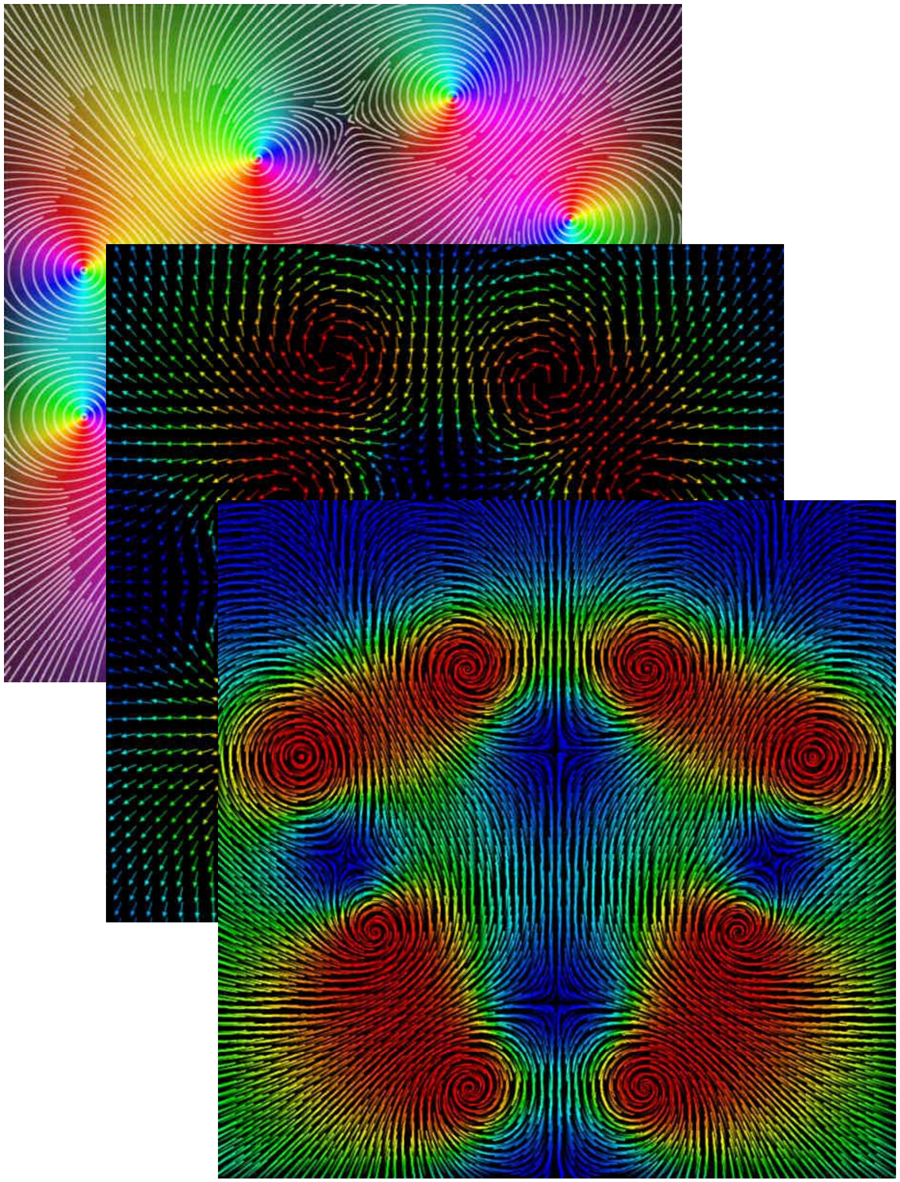
Designing Valid Empirical Experiments

- Experimental Validity
  - Does experiment really measure what we want it to measure?
  - Do our results really mean what we think (and hope) they mean?
  - Are our results reliable?
    - If we run the experiment again, will we get the same results?
    - Will others get the same results?
- Validity is a large topic in empirical inquiry

Example of a Validity Issue



- **2D Flow Visualization Experiment**
- **Tested different visualization methods**
- **Measured subjects' ability to locate critical points**
  - **error, response time**
- **Validity Issue:**
  - **Interested in which visualization method is most effective**
  - **How well does what we measured relate to "effectiveness?"**

Experimental Variables

- **Independent Variables**
  - What the experiment is studying
  - Occur at different levels
    - Example: stereopsis, at the levels of stereo, mono
  - Systematically varied by experiment

- **Dependent Variables**
  - What the experiment measures
  - Assume dependent variables will be effected by independent variables
  - Must be measurable quantities
    - Time, task completion counts, error counts, survey answers, scores, etc.
    - Example: VR navigation performance, in total time

Experimental Variables

- Independent variables can vary in two ways
  - Between-subjects: each subject sees a different level of the variable
    - Example: ½ of subjects see stereo, ½ see mono
  - Within-subjects: each subject sees all levels of the variable
    - Example: each subject sees both stereo and mono
- Confounding factors (or confounding variables)
  - Factors that are not being studied, but will still affect experiment
    - Example: stereo condition less bright than mono condition
  - Important to predict and control confounding factors, or experimental validity will suffer

Experimental Design

- **Empiricism**
- **Experimental Validity**
- ***Experimental Design***
- **Gathering Data**
- **Describing Data**
  - **Graphing Data**
  - **Descriptive Statistics**
- **Inferential Statistics**
  - **Hypothesis Testing**
  - **Hypothesis Testing Means**
  - **Power**
  - **Analysis of Variance and Factorial Experiments**

Experimental Designs

- **2 x 1 is simplest possible design, with one independent variable at two levels:**

Variable
level 1
level 2

Stereopsis
stereo
mono

- **Important confounding factors for within subject variables:**
  - **Learning effects**
  - **Fatigue effects**
- **Control these by counterbalancing the design**
  - **Ensure no systematic variation between levels and the order they are presented to subjects**

Subjects	1st condition	2nd condition
1, 3, 5, 7	stereo	mono
2, 4, 6, 8	mono	stereo

Factorial Designs

- *n* x 1 designs generalize the number of levels:

VE terrain type
flat
hilly
mountainous

- Factorial designs generalize number of independent variables and the number of levels of each variable
- Examples: *n* x *m* design, *n* x *m* x *p* design, etc.
- Must watch for factorial explosion of design size!

3 x 2 design:

	Stereopsis	
VE terrain type	stereo	mono
flat		
hilly		
mountainous		

Cells and Repetitions

- **Cell**: each combination of levels
- **Repetitions**: typically, the combination of levels at each cell is repeated a number of times

	Stereopsis	
VE terrain type	stereo	mono
flat		
hilly		
mountainous		

cell

- Example of how this design might be described:
  - "A 3 (VE terrain type) by 2 (stereopsis) within-subjects design, with 4 repetitions of each cell."
  - This means each subject would see 3 x 2 x 4 = 24 total conditions
  - The presentation order would be counterbalanced

Counterbalancing

- Addresses time-based confounding factors:
  - Within-subjects variables: control learning and fatigue effects
  - Between-subjects variables: control calibration drift, weather, other factors that vary with time
- There are two counterbalancing methods:
  - Random permutations
  - Systematic variation
    - Latin squares are a very useful and popular technique

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

2 x 2

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \\ 3 & 1 & 4 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

4 x 4

- Latin square properties:
  - Every level appears in every position the same number of times
  - Every level is followed by every other level
  - Every level is preceded by every other level

6 x 3 (there is no 3 x 3 that has all 3 properties)

Counterbalancing Example

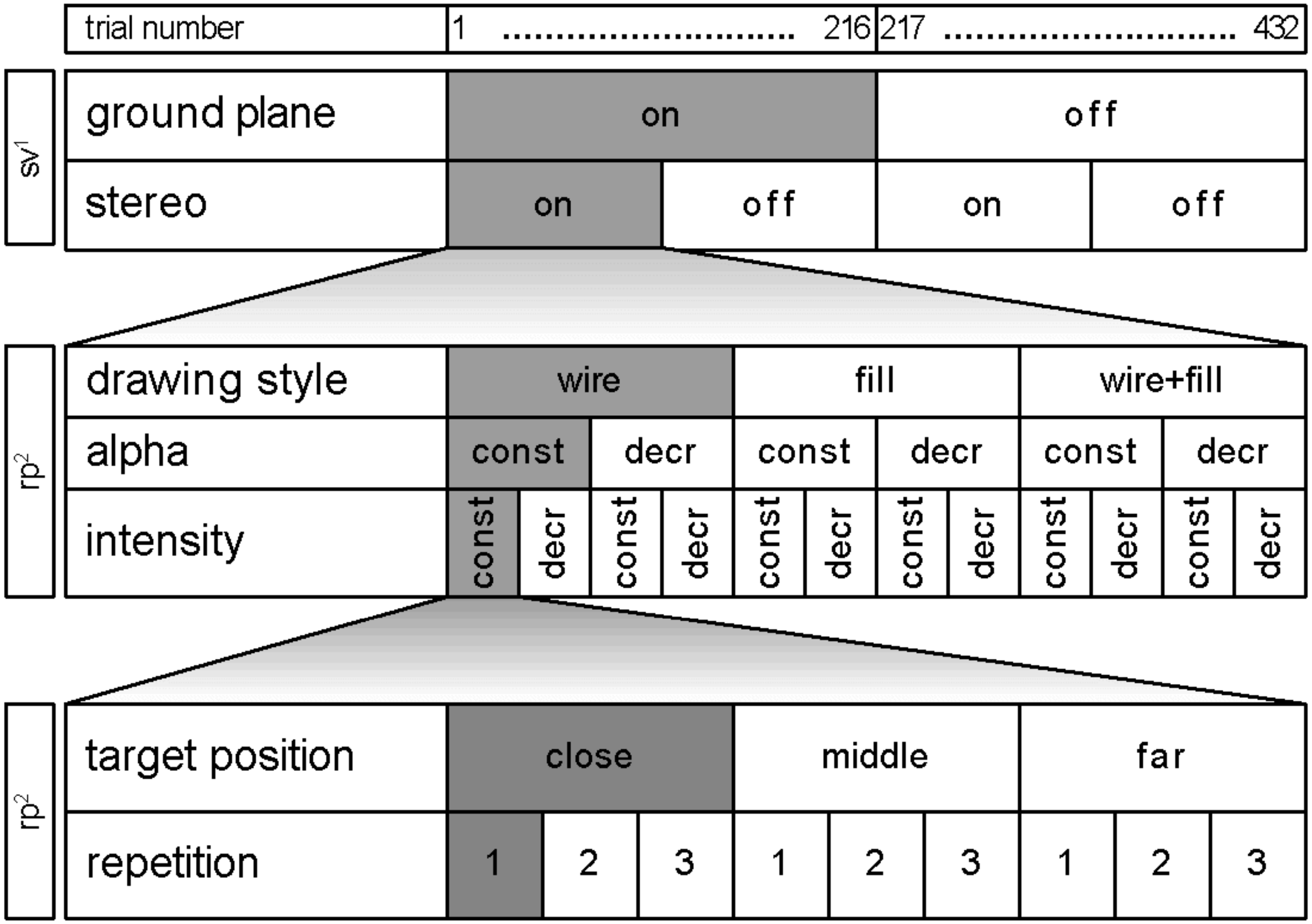
- "A 3 (VE terrain type) by 2 (stereopsis) within-subjects design, with 4 repetitions of each cell."
- Form Cartesian product of Latin squares {6 x 3} (VE Terrain Type) ⊗ {2 x 2} (Stereopsis)
- Perfectly counterbalances groups of 12 subjects

Subject	Presentation Order
1	1A, 1B, 2A, 2B, 3A, 3B
2	1B, 1A, 2B, 2A, 3B, 3A
3	2A, 2B, 3A, 3B, 1A, 1B
4	2B, 2A, 3B, 3A, 1B, 1A
5	3A, 3B, 1A, 1B, 2A, 2B
6	3B, 3A, 1B, 1A, 2B, 2A
7	1A, 1B, 3A, 3B, 2A, 2B
8	1B, 1A, 3B, 3A, 2B, 2A
9	2A, 2B, 1A, 1B, 3A, 3B
10	2B, 2A, 1B, 1A, 3B, 3A
11	3A, 3B, 2A, 2B, 1A, 1B
12	3B, 3A, 2B, 2A, 1B, 1A

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ B & A \end{bmatrix}$$

Experimental Design Example #1



[1] sv = systemically varied, [2] rp = randomly permuted

- **All variables within-subject**

From [Living et al. 03]

Experimental Design Example #2

Between Subject	Stereo Viewing		*on*				*off*			
	Control Movement		*rate*		*position*		*rate*		*position*	
	Frame of Reference		*ego*	*exo*	*ego*	*exo*	*ego*	*exo*	*ego*	*exo*
Within Subject	Computer Platform	*cave*	subjects 1 – 4	subjects 5 – 8	subjects 9 – 12	subjects 13 – 16	subjects 17 – 20	subjects 21 – 24	subjects 25 – 28	subjects 29 – 32
		wall								
		workbench								
		desktop								

- **Mixed design: some variables between-subject, others within-subject.**

From [Swan et al. 03]

Gathering Data

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Dependent Measures

- **Workhorse measures:**
  - Response time, error counts
- **Some additional measures (many others exist):**
  - Critical incidents
  - 6 degree-of-freedom tracker trajectory (head, hand)
  - Eye-tracker data
  - Answers scored by experts
  - Questions answered on Likert scale:

I was able to generate a visualization that tested my hypothesis:				
strongly agree	Agree	neutral	disagree	strongly disagree

Cognitive Analysis Techniques

- Cognitive techniques may yield important insights for Visualization analysis

- Example of a cognitive analysis:
  - Subject uses think out loud protocol
  - Session videotaped, perhaps logged
  - Log is divided into brief intervals
  - Each interval labeled with cognitive state
  - Counts of cognitive states are analyzed

- But cognitive techniques give more qualitative (less quantitative) results

Cognitive analysis from [Wu et al. 96]

Pilot Testing a Design

- **Experimental designs have to be tested and iterated (debugged)**
- **Typical flow:**
  - **1st run: subjects are you, collaborators**
  - **2nd run: small number of preliminary subjects**
  - **3rd run: subset of real subjects**
- **With each run, problems are revealed; fix problems and iterate**
- **For later runs, perform data analysis before gathering additional data**

Graphing Data

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Types of Statistics

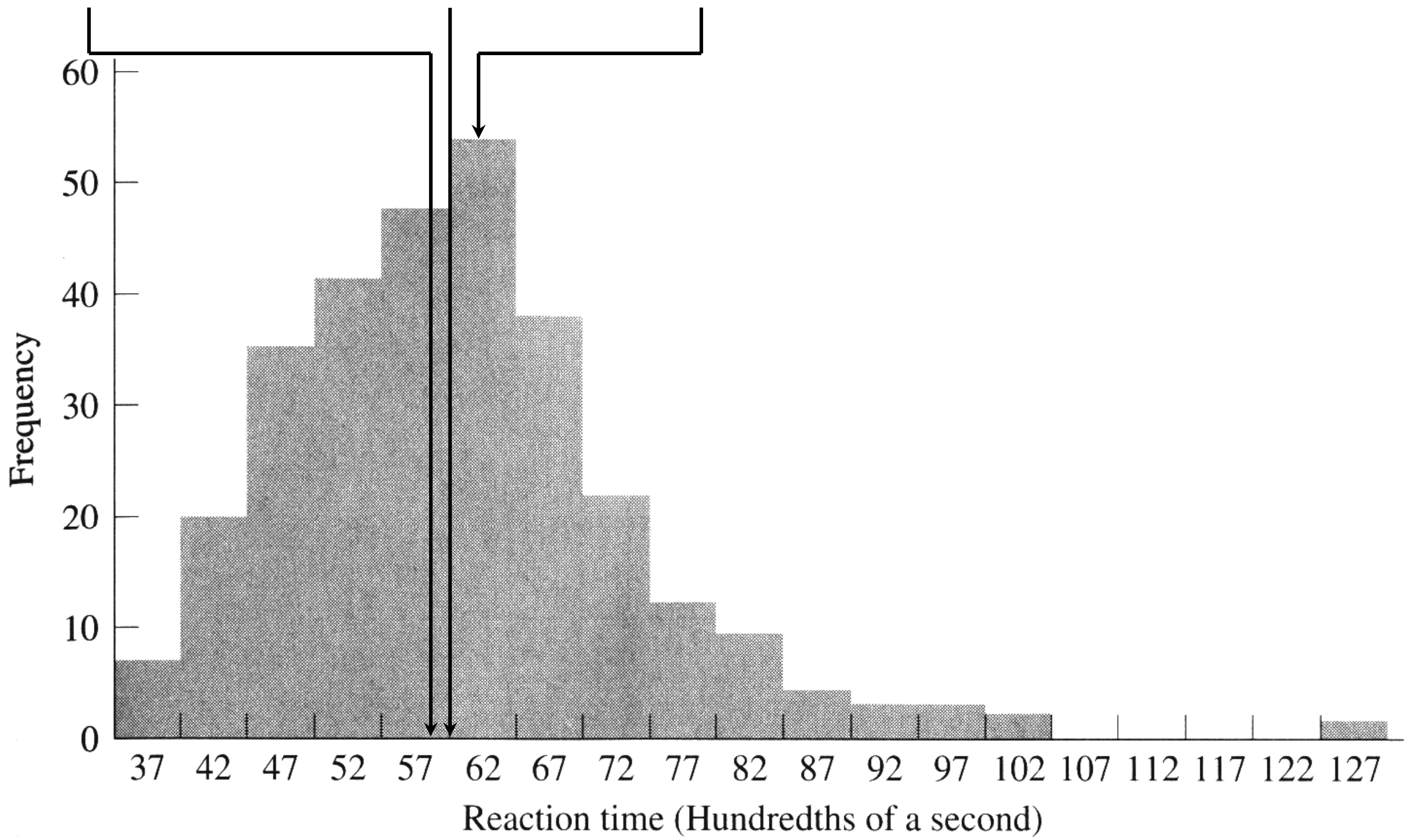
- **Descriptive Statistics**
  - Describe and explore data
  - Summary statistics:
    many numbers $\rightarrow$ few numbers
  - All types of graphs and visual representations
  - Data analysis begins with descriptive stats
    - Understand data distribution
    - Test assumptions of significance tests

- **Inferential Statistics**
  - Detect relationships in data
  - Significance tests
  - Infer population characteristics from sample characteristics

Exploring Data with Graphs

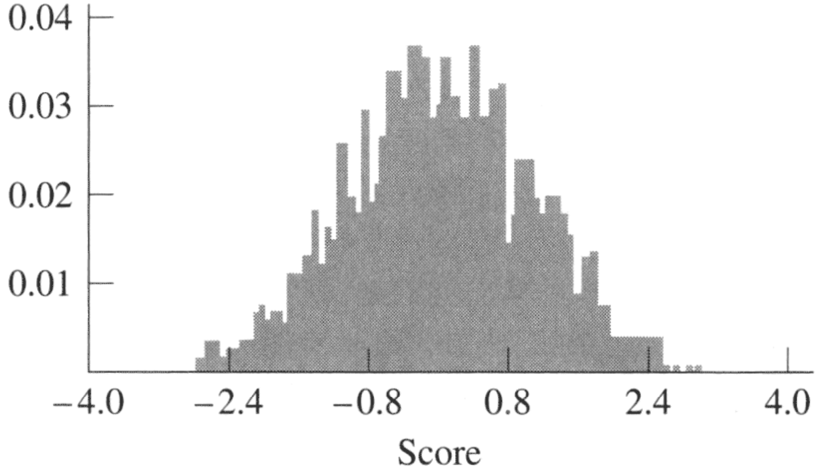
- **Histogram common data overview method**

median = 59.5 mean = 60.26 mode = 62

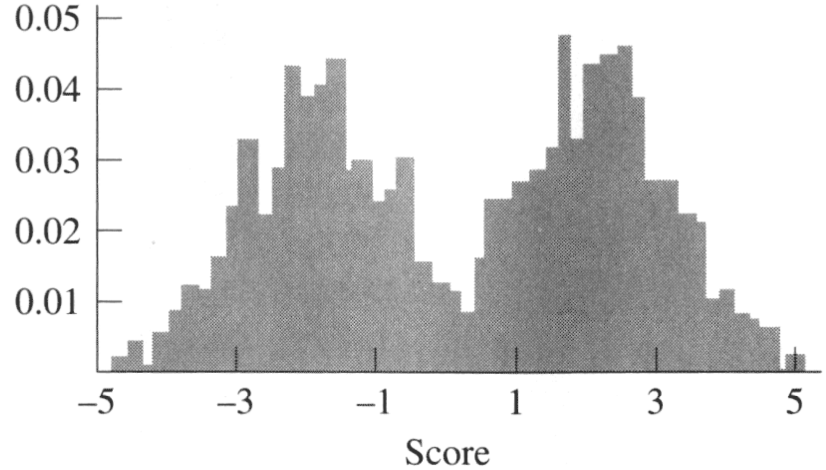


From [Howell 02] p 21

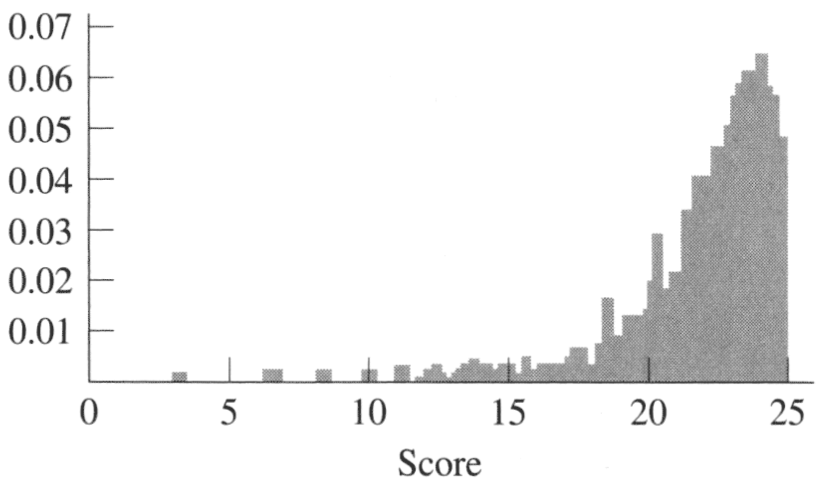
Classifying Data with Histograms



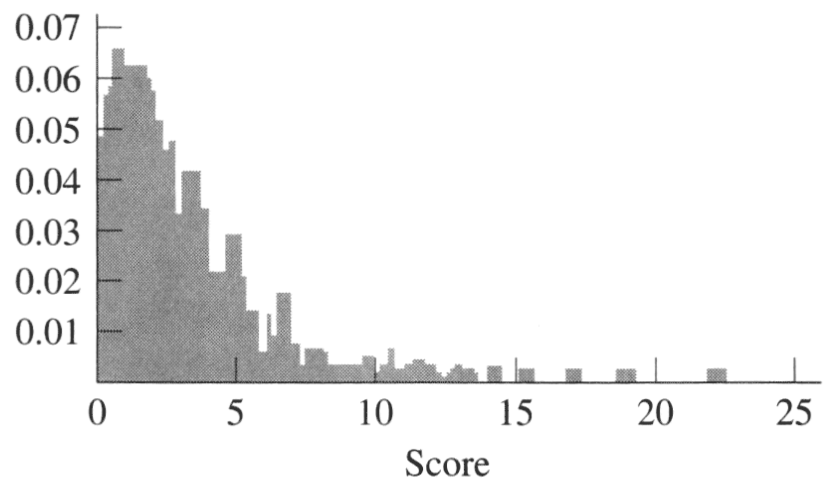
(a) Normal



(b) Bimodal



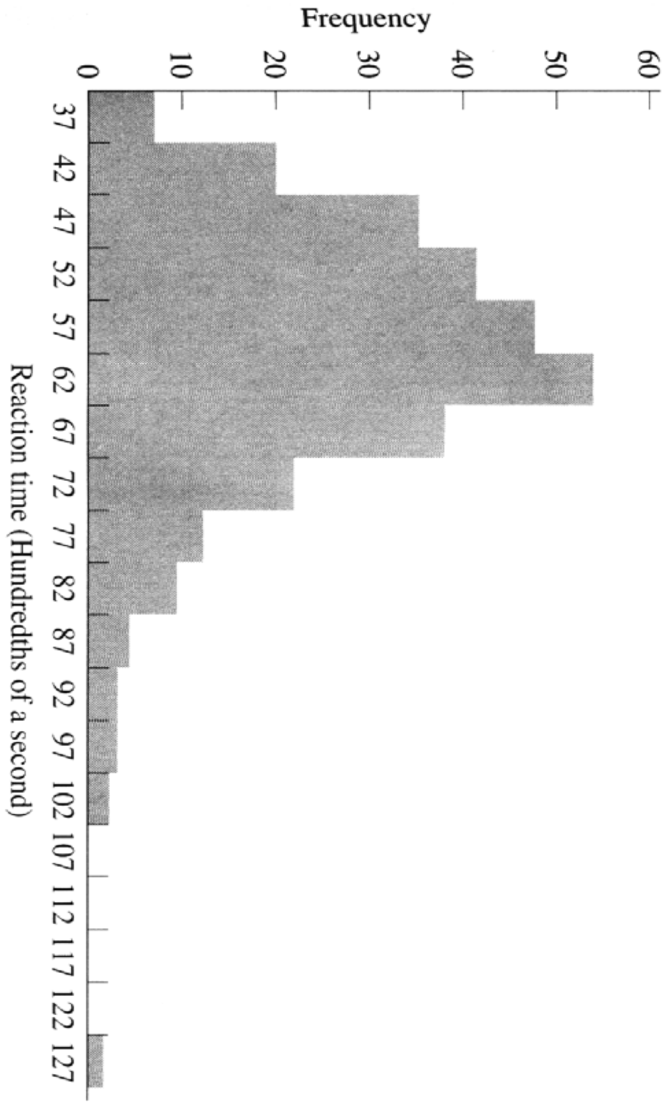
(c) Negatively skewed



(d) Positively skewed

From [Howell 02] p 28

Stem-and-Leaf:
Histogram From Actual Data



Raw Data	Stem	Leaf
36 37 38 38 39 39 39 40	3s	67
40 40 40 41 41 41 42 42	3.	88999
42 43 43 43 43 43 44 44	4*	0000111
44 44 44 45 45 45 45 45	4t	22233333
45 46 46 46 46 46 46 46	4f	44444555555
46 46 46 46 47 47 47 47	4s	66666666666777777777
47 47 47 47 47 48 48 48	4.	888899999
48 49 49 49 49 49 50 50	5*	00000111111111111
50 50 50 51 51 51 51 51	5t	222222222233333333
51 51 51 51 51 51 51 52	5f	4444445555555
52 52 52 52 52 52 52 52	5s	66666666667777777
52 53 53 53 53 53 53 53	5.	88888888888899999999999
53 54 54 54 54 54 54 55	6*	00000000000011111111111
55 55 55 55 55 55	6t	222222222222223333333333
	6f	444444455555555
	6s	6666666677777777777777
	6.	889999999
	7*	01111
	7t	2222222333
	7f	44444455
	7s	666677
	7.	88899
	8*	00011
	8t	2333
	8f	5
	8s	67
	8.	8
	9*	0
	9t	
	9f	4455
	9s	
	9.	8
	High	104; 104; 125

FIGURE 2.4 *Stem-and-leaf display for reaction time data*

From [Howell 02] p 21, 23

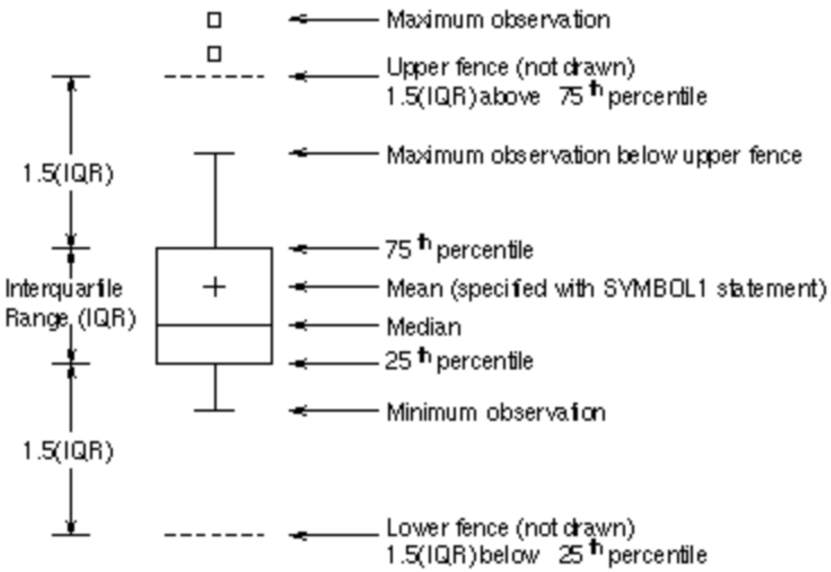
Stem-and-Leaf: Histogram From Actual Data

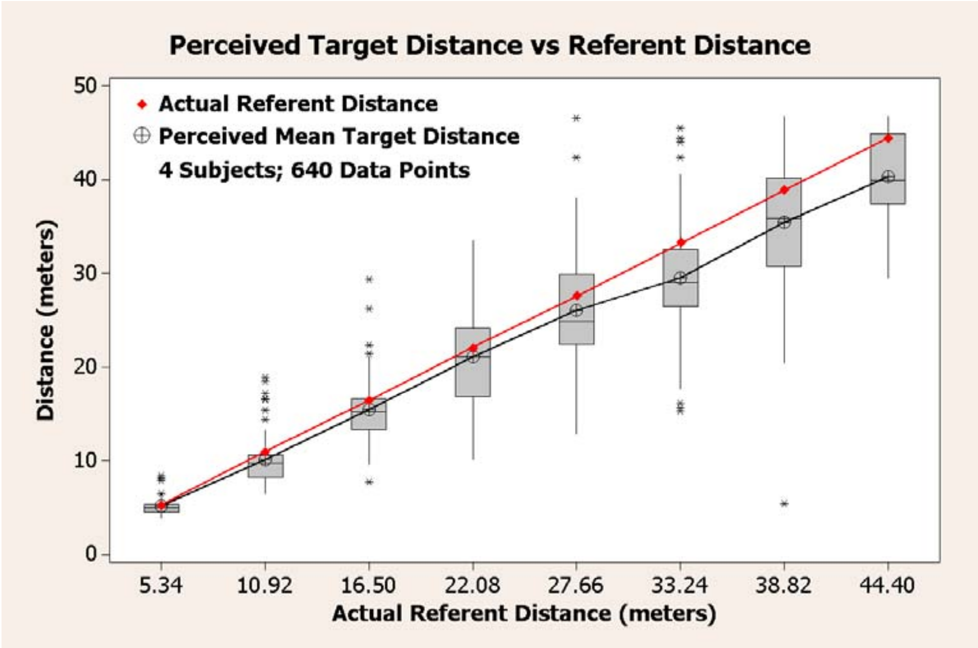
Midterm #1 Grades

0	0%	F	0	
0	0%	F	1	
0	0%	F	2	
0	0%	F	3	
1	3%	F	4	7
0	0%	F	5	
4	12%	D	6	1789
3	9%	C	7	024
10	30%	B	8	0014458889
14	42%	A	9	01112333455579
0	0%	A	10	
32				

Grades from my fall 2007 Analysis of Algorithms class; first midterm

Boxplot

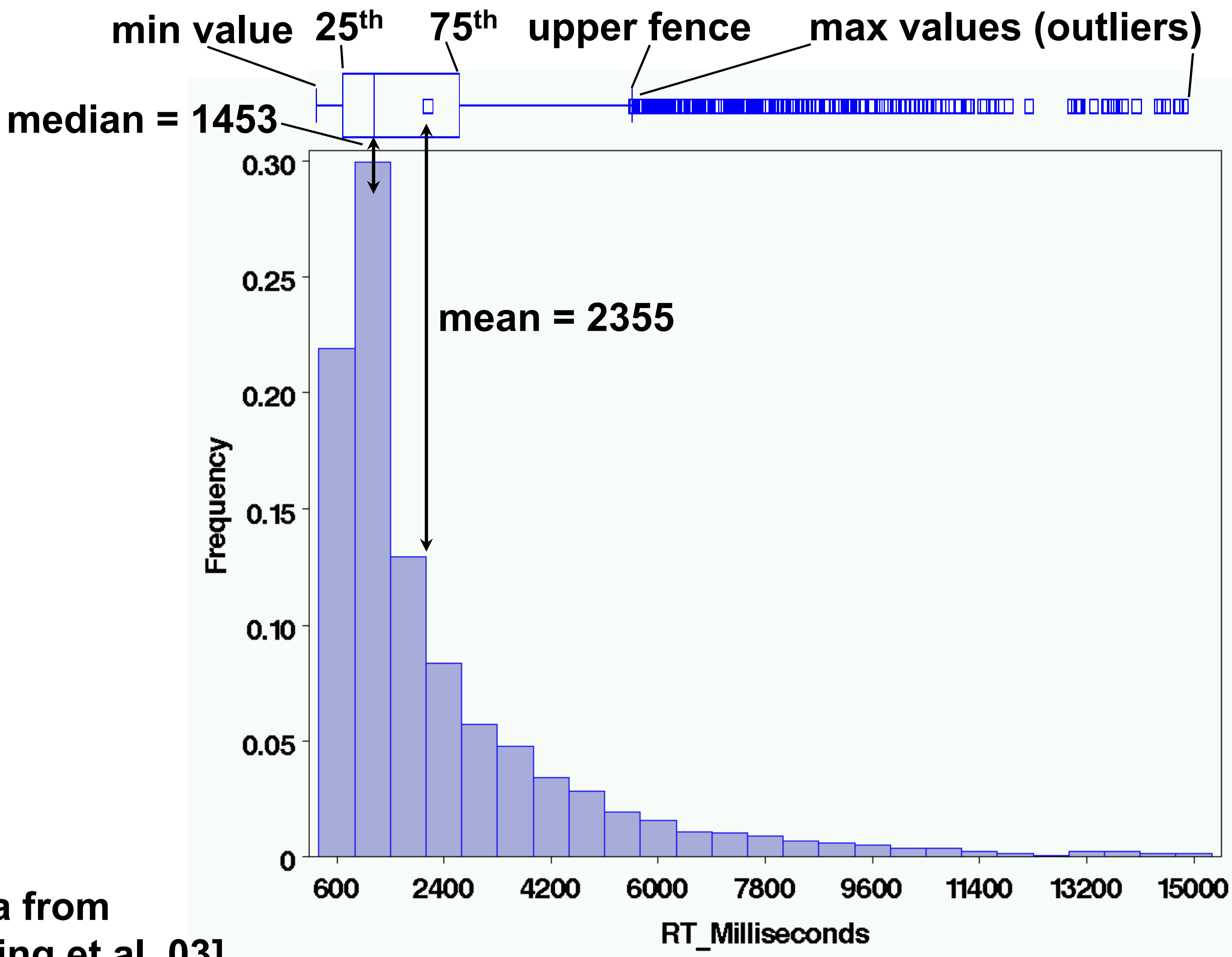




- **Emphasizes variation and relationship to mean**
- **Because narrow, can be used to display side-by-side groups**

Data from [Swan et al. 06]

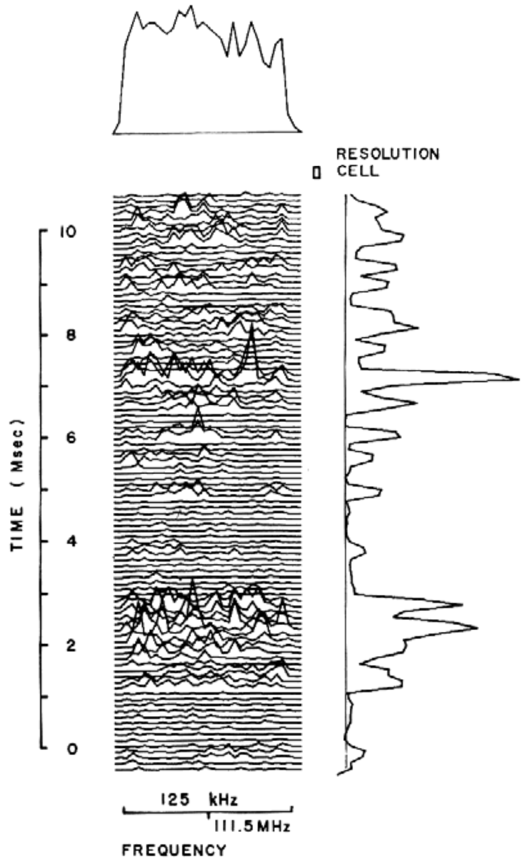
Example Histogram and Boxplot from Real Data



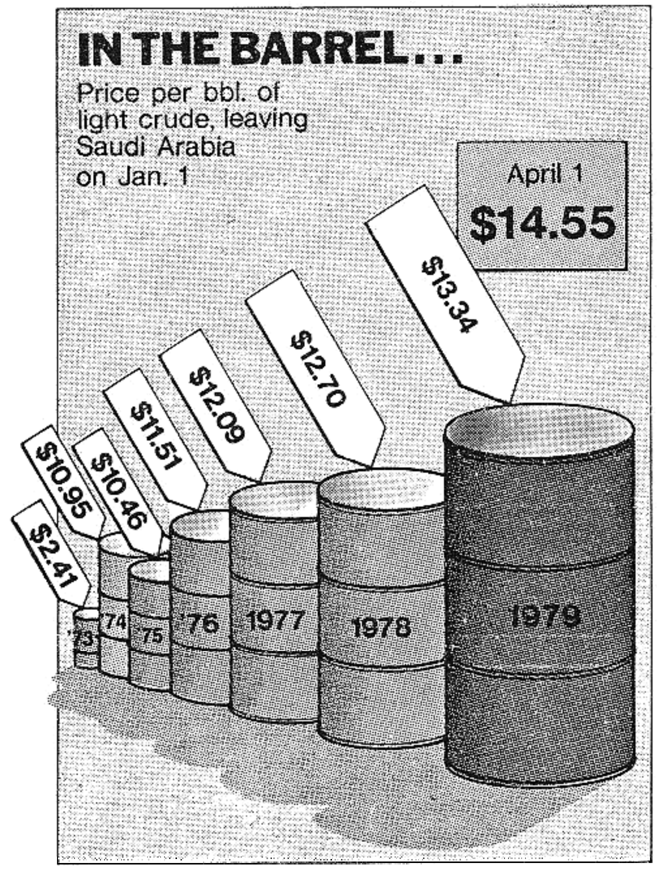
Data from
[Living et al. 03]

We Have Only Scratched the Surface…

- **There are a vary large number of graphing techniques**
- **Tufte's [83, 90] works are classic, and stat books show many more examples (e.g. Howell [03]).**



Lots of good examples…



And plenty of bad examples!

From [Tufte 83], p 134, 62

Descriptive Statistics

- Empiricism
- Experimental Validity
- Usability Engineering
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Summary Statistics

- **Many numbers → few numbers**

- **Measures of central tendency:**
  - **Mean: average**
  - **Median: middle data value**
  - **Mode: most common data value**

- **Measures of variability / dispersion:**
  - **Mean absolute deviation**
  - **Variance**
  - **Standard Deviation**

Populations and Samples

- **Population:**
  - Set containing every possible element that we want to measure
  - Usually a Platonic, theoretical construct
  - Mean: μ Variance: σ^2 Standard deviation: σ

- **Sample:**
  - Set containing the elements we actually measure (our subjects)
  - Subset of related population
  - Mean: $\bar{X}$ Variance: s^2 Standard deviation: s
    Number of samples: N

Measuring Variability / Dispersion

Mean:

$$\overline{X} = \frac{\sum X}{N}$$

Mean absolute deviation:

$$\text{m.a.d.} = \frac{\sum \left| X - \overline{X} \right|}{N}$$

Variance:

$$s^2 = \frac{\sum \left(X - \overline{X} \right)^2}{N - 1}$$

$$\sigma^2 = \frac{\sum \left(X - \mu \right)^2}{N}$$

Standard deviation:

$$s = \sqrt{\frac{\sum \left(X - \overline{X} \right)^2}{N - 1}}$$

- **Standard deviation uses same units as samples and mean.**
- **Calculation of population variance σ^2 is theoretical, because μ almost never known and the population size *N* would be very large (perhaps infinity).**

Sums of Squares, Degrees of Freedom, Mean Squares

- **Very common terms and concepts:**

$$s^2 = \frac{\sum \left(X - \overline{X}\right)^2}{N-1} = \frac{\text{SS}}{df} = \frac{\text{sums of squares}}{\text{degrees of freedom}} = \text{MS (mean squares)}$$

- **Sums of squares:**
  - **Summed squared deviations from mean**
- **Degrees of freedom:**
  - **Given a set of *N* observations used in a calculation, how many numbers in the set may vary**
  - **Equal to *N* minus number of means calculated**
- **Mean squares:**
  - **Sums of squares divided by degrees of freedom**
  - **Another term for variance, used in ANOVA**

Example: Degrees of Freedom

- Samples: 6, 8, 10; N = 3; X = 8

- If mean must remain X = 8;
  how many numbers may vary?

- Answer: 2 may vary (2 + 36 + a)/3 = 8
  - Value of a constrained to keep X = 8

- We say that this set has
  N – 1 = 2 degrees of freedom (*dof*, *df*)
  - Generally equal to N minus 1 per mean calculated

Hypothesis Testing

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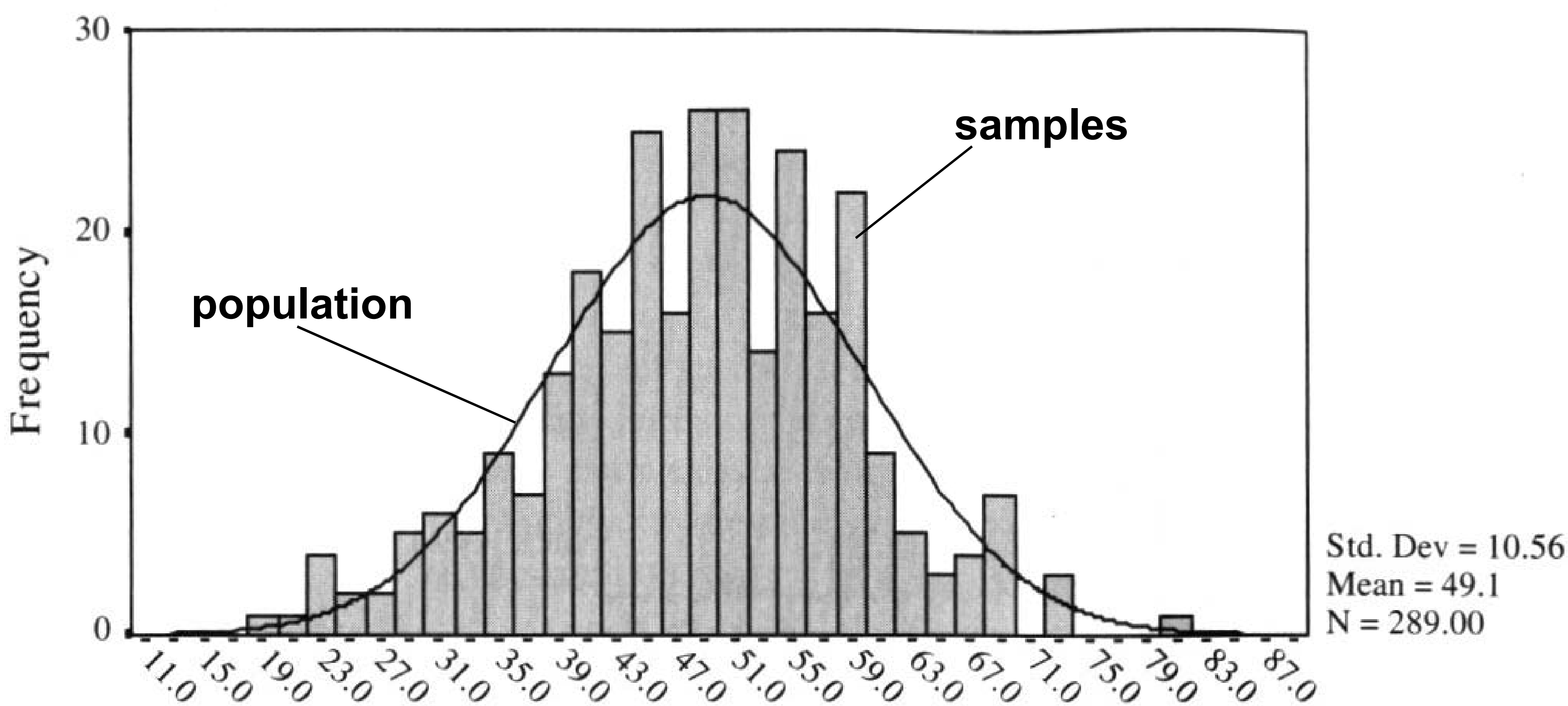
Hypothesis Testing

- **Goal is to infer population characteristics from sample characteristics**



From [Howell 02], p 78

Testable Hypothesis

- **General hypothesis: The research question that motivates the experiment.**

- **Testable hypothesis: The research question expressed in a way that can be measured and studied.**

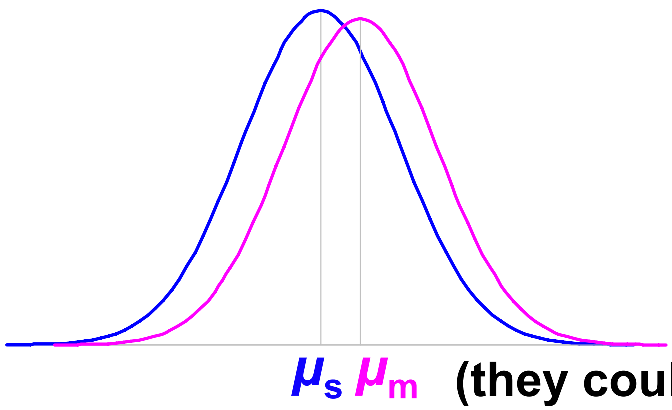
- **Generating a good testable hypothesis is a real skill of experimental design.**
  - **By *good*, we mean contributes to experimental validity.**
  - **Skill best learned by studying and critiquing previous experiments.**

Testable Hypothesis Example

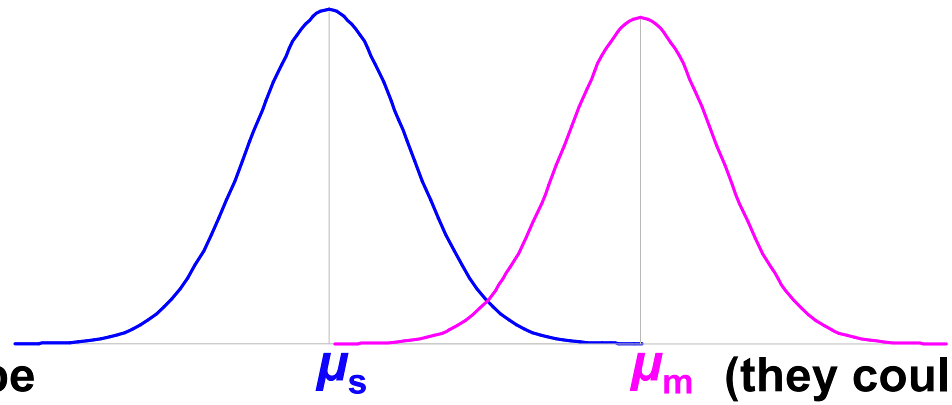
- **General hypothesis**: Stereo will make people more effective when navigating through a virtual environment (VE).
- **Testable hypothesis**: We measure time it takes for subjects to navigate through a particular VE, under conditions of stereo and mono viewing. We hypothesis subjects will be faster under stereo viewing.
- Testable hypothesis requires a measurable quantity:
  - Time, task completion counts, error counts, etc.
- Some factors effecting experimental validity:
  - Is VE representative of something interesting (e.g., a real-world situation)?
  - Is navigation task representative of something interesting?
  - Is there an underlying theory of human performance that can help predict the results? Could our results contribute to this theory?

What Are the Possible Alternatives?

- Let time to navigate be μ_s: stereo time; μ_m: mono time
  - Perhaps there are two populations: $\mu_s - \mu_m = d$

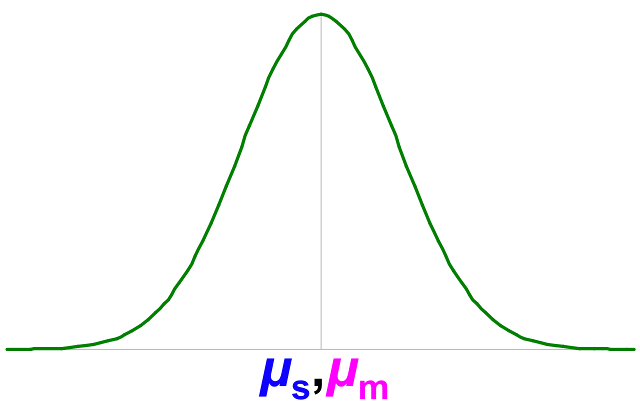


(they could be close together)



(they could be far apart)

  - Perhaps there is one population: $\mu_s - \mu_m = 0$



Hypothesis Testing Procedure

1. Develop testable hypothesis H_1: $\mu_s - \mu_m = d$
   - (E.g., subjects faster under stereo viewing)

2. Develop null hypothesis H_0: $\mu_s - \mu_m = 0$
   - Logical opposite of testable hypothesis

3. Construct sampling distribution assuming H_0 is true.

4. Run an experiment and collect samples; yielding sampling statistic X.
   - (E.g., measure subjects under stereo and mono conditions)

5. Referring to sampling distribution, calculate conditional probability of seeing X given H_0: $p(X \mid H_0)$.
   - If probability is low ($p \leq 0.05$, $p \leq 0.01$), we are unlikely to see X when H_0 is true. We reject H_0, and embrace H_1.
   - If probability is not low ($p > 0.05$), we are likely to see X when H_0 is true. We do not reject H_0.

Example 1: VE Navigation with Stereo Viewing

1. Hypothesis H_1: $\mu_s - \mu_m = d$
   - Subjects faster under stereo viewing.
2. Null hypothesis H_0: $\mu_s - \mu_m = 0$
   - Subjects same speed whether stereo or mono viewing.
3. Constructed sampling distribution assuming H_0 is true.
4. Ran an experiment and collected samples:
   - 32 subjects, collected 128 samples
   - X_s = 36.431 sec; X_m = 34.449 sec; $X_s - X_m$ = 1.983 sec
5. Calculated conditional probability of seeing 1.983 sec given H_0: p(1.983 sec | H_0) = 0.445.
   - p = 0.445 not low, we are likely to see 1.983 sec when H_0 is true. We do not reject H_0.
   - This experiment did not tell us that subjects were faster under stereo viewing.

Data from [Swan et al. 03]

Example 2: Effect of Intensity on AR Occluded Layer Perception

1. Hypothesis H_1: $\mu_c - \mu_d = d$
   - Tested constant and decreasing intensity. Subjects faster under decreasing intensity.
2. Null hypothesis H_0: $\mu_c - \mu_d = 0$
   - Subjects same speed whether constant or decreasing intensity.
3. Constructed sampling distribution assuming H_0 is true.
4. Ran an experiment and collected samples:
   - 8 subjects, collected 1728 samples
   - X_c = 2592.4 msec; X_d = 2339.9 msec; $X_c - X_d$ = 252.5 msec
5. Calculated conditional probability of seeing 252.5 msec given H_0: p(252.5 msec | H_0) = 0.008.
   - p = 0.008 is low ($p \leq 0.01$); we are unlikely to see 252.5 msec when H_0 is true. We reject H_0, and embrace H_1.
   - This experiment suggests that subjects are faster under decreasing intensity.

Data from [Living et al. 03]

Some Considerations…

- The conditional probability $p(X \mid H_0)$
  - Much of statistics involves how to calculate this probability; source of most of statistic's complexity
  - Logic of hypothesis testing the same regardless of how $p(X \mid H_0)$ is calculated
  - If you can calculate $p(X \mid H_0)$, you can test a hypothesis

- The null hypothesis H_0
  - H_0 usually in form $f(\mu_1, \mu_2,\ldots) = 0$
  - Gives hypothesis testing a double-negative logic: assume H_0 as the opposite of H_1, then reject H_0
  - Philosophy is that can never prove something true, but can prove it false
  - H_1 usually in form $f(\mu_1, \mu_2,\ldots) \neq 0$; we don't know what value it will take, but main interest is that it is not 0

When We Reject H_0

- Calculate $\alpha = p(X \mid H_0)$, when do we reject H_0?
  - In psychology, two levels: $\alpha \leq 0.05$; $\alpha \leq 0.01$
  - Other fields have different values
- What can we say when we reject H_0 at $\alpha = 0.008$?
  - "If H_0 is true, there is only an 0.008 probability of getting our results, and this is unlikely."
    - Correct!
  - "There is only a 0.008 probability that our result is in error."
    - Wrong, this statement refers to $p(H_0)$, but that's not what we calculated.
  - "There is only a 0.008 probability that H_0 could have been true in this experiment."
    - Wrong, this statement refers to $p(H_0 \mid X)$, but that's not what we calculated.

When We Don't Reject H_0

- What can we say when we don't reject H_0 at $\alpha = 0.445$?
  - "We have proved that H_0 is true."
  - "Our experiment indicates that H_0 is true."
    - Wrong, statisticians agree that hypothesis testing cannot prove H_0 is true.
- Statisticians do not agree on what failing to reject H_0 means.
  - Conservative viewpoint (Fisher):
    - We must suspend judgment, and cannot say anything about the truth of H_0.
  - Alternative viewpoint (Neyman & Pearson):
    - We "accept" H_0, and act as if it's true for now…
    - But future data may cause us to change our mind

From [Howell 02], p 99

Probabilistic Reasoning

- **If hypothesis testing was absolute:**
  - If H_0 is true, then *X* cannot occur…however, *X* has occurred…therefore H_0 is false.
  - e.g.: If a person is a Martian, then they are not a member of Congress (true)…this person is a member of Congress…therefore they are not a Martian. (correct result)
  - e.g.: If a person is an American, then they are not a member of Congress (false)…this person is a member of Congress…therefore they are not an American. (correct result because if-then false)

- **However, hypothesis testing is probabilistic:**
  - If H_0 is true, then *X* is highly unlikely…however, *X* has occurred…therefore H_0 is highly unlikely.
  - e.g.: If a person is an American, then they are probably not a member of Congress (true, right?)…this person is a member of Congress…therefore they are probably not an American. (correct hypothesis testing reasoning, but incorrect result)

From [Cohen 94]

Hypothesis Testing Outcomes

		Decision	
		Reject H_0	Don't reject H_0
True state of the world	H_0 false	correct a result! $p = 1 - \beta$ = power	wrong type II error $p = \beta$
	H_0 true	wrong type I error $p = \alpha$	correct (but wasted time) $p = 1 - \alpha$

- $p(X \mid H_0)$ compared to α, so hypothesis testing involves setting α (typically 0.05 or 0.01)
- Two ways to be right:
  - Find a result
  - Fail to find a result and waste time running an experiment
- Two ways to be wrong:
  - Type I error: we think we have a result, but we are wrong
  - Type II error: a result was there, but we missed it

When Do We *Really* Believe a Result?

- When we reject H_0, we have a result, but:
  - It's possible we made a type I error
  - It's possible our finding is not reliable
    - Just an artifact of our particular experiment
- So when do we *really* believe a result?
  - Statistical evidence
    - α level: ($p < .05$, $p < .01$, $p < .001$)
    - Power
  - Meta-statistical evidence
    - Plausible explanation of observed phenomena
      - Based on theories of human behavior: perceptual, cognitive psychology, control theory, etc.
    - Repeated results
      - Especially by others

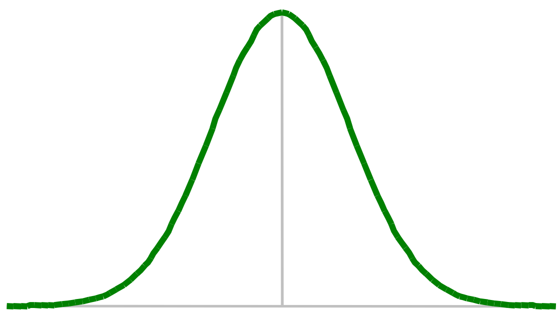
Hypothesis Testing Means

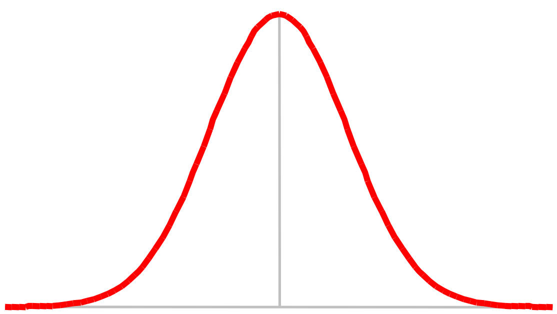
- Empiricism
- Experimental Validity
- Experimental Design
- Gathering Data
- Describing Data
  - Graphing Data
  - Descriptive Statistics
- Inferential Statistics
  - Hypothesis Testing
  - *Hypothesis Testing Means*
  - Power
  - Analysis of Variance and Factorial Experiments

Hypothesis Testing Means

- How do we calculate $\alpha = p(X \mid H_0)$, when *X* is a mean?
  - Calculation possible for other statistics, but most common for means

- Answer: we refer to a sampling distribution
- We have two conceptual functions:
  - Population: unknowable property of the universe
  - Distribution: analytically defined function, has been found to match certain population statistics

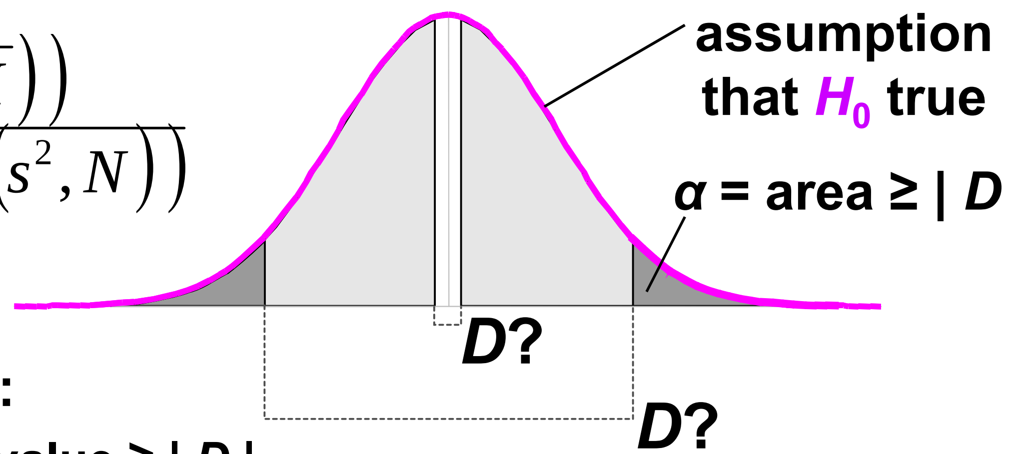




Calculating $\alpha = p(X \mid H_0)$ with A Sampling Distribution

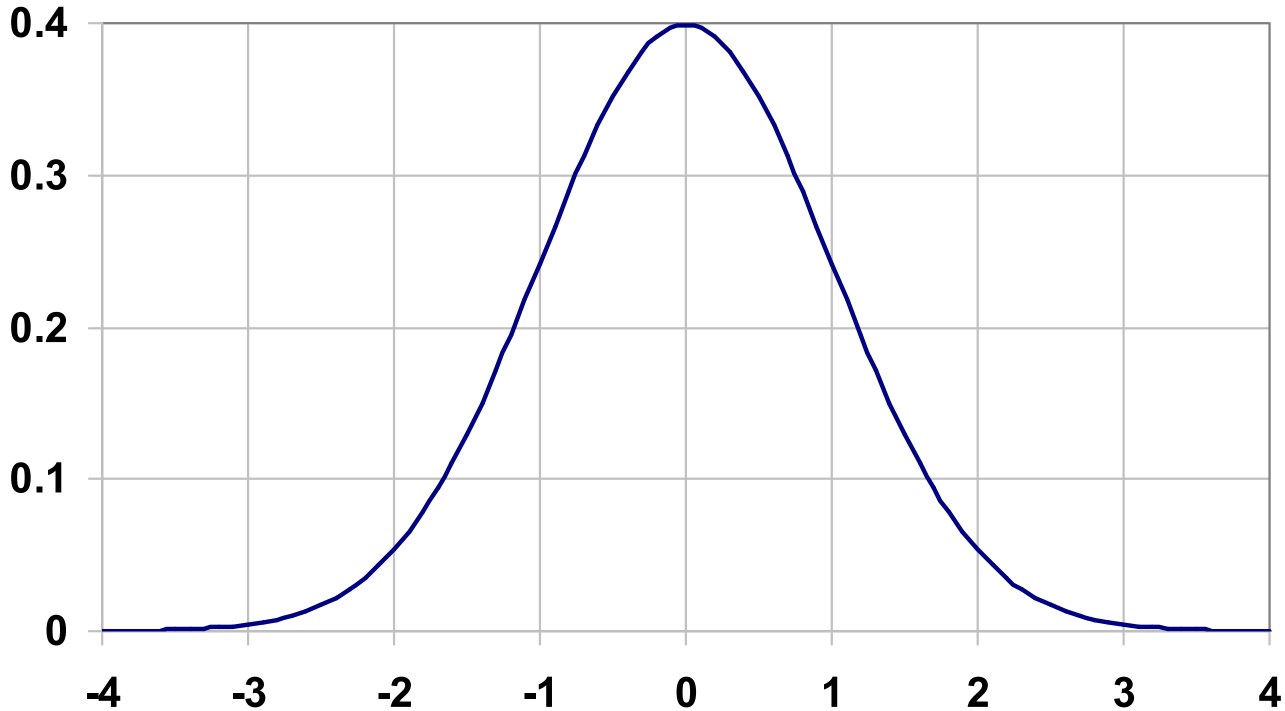
- **Sampling distributions are analytic functions with area 1**
- **To calculate $\alpha = p(X \mid H_0)$ given a distribution, we first calculate the value *D*, which comes from an equation of the form:**

$$D = \frac{\left(\text{size of effect} : f\left(\overline{X}\right)\right)}{\left(\text{variability of effect} : f\left(s^2, N\right)\right)}$$



- **$\alpha = p(X \mid H_0)$ is equal to:**
  - **Probability of seeing a value ≥ | *D* |**
  - **2 * (area of the distribution to the right of | *D* |)**
- **If H_0 true, we expect *D* to be near central peek of distribution**
- **If *D* far from central peek, we have reason to reject the idea that H_0 is true**

A Distribution for Hypothesis Testing Means



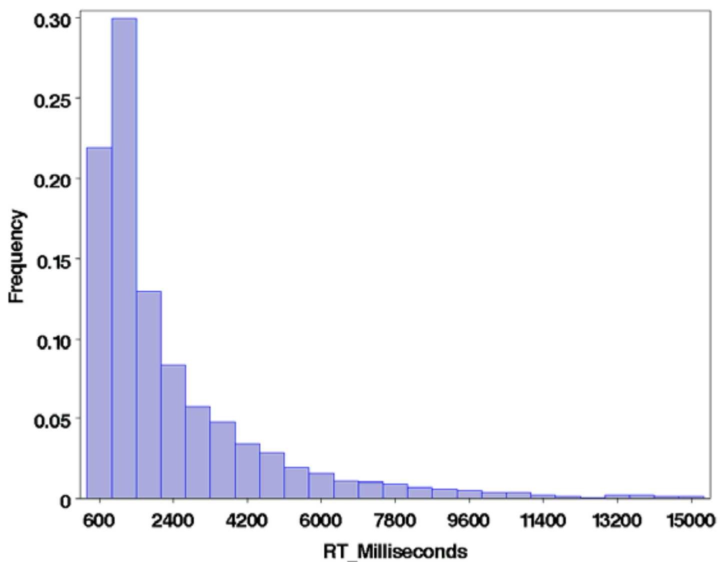
- **The Standard Normal Distribution (μ = 0, σ = 1) (also called the *Z*-distribution):**

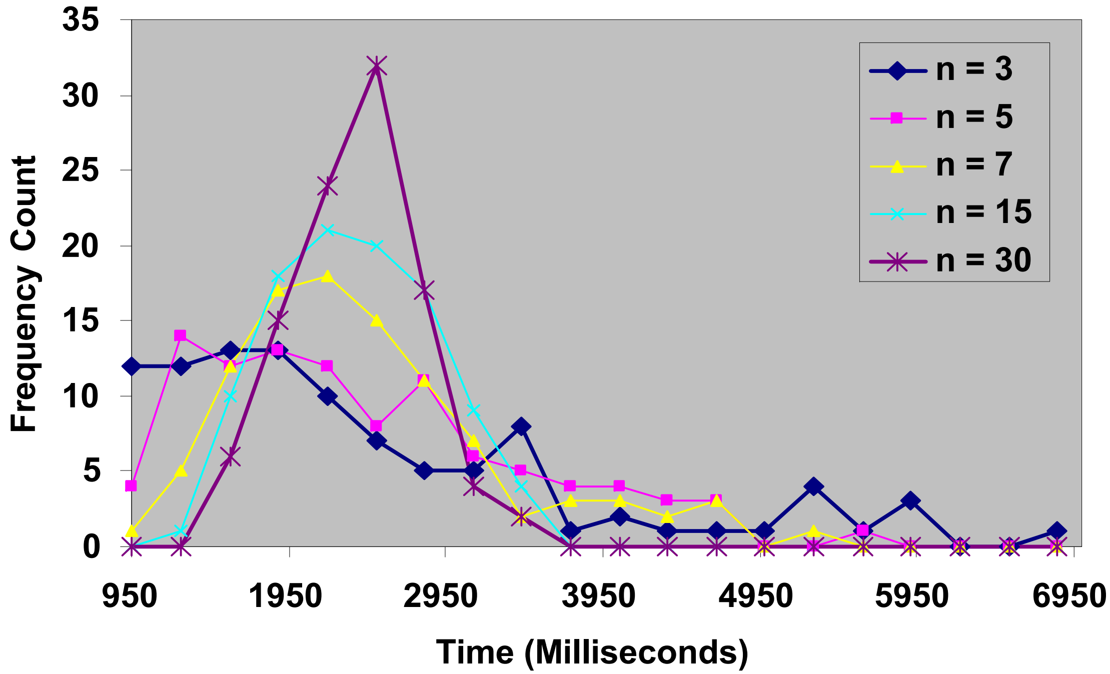
$$N(X;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(X-\mu)^2}{2\sigma^2}}$$

The Central Limit Theorem

- **Full Statement:**
  - Given population with (μ, σ^2), the sampling distribution of means drawn from this population is distributed (μ, σ^2/n), where n is the sample size. As n increases, the sampling distribution of means approaches the normal distribution.
- **Implication:**
  - As n increases, distribution of means becomes normal, regardless of how "non-normal" the population looks.
- **How big does n have to be before means look normally distributed?**
  - For very "non-normal" data, $n \approx 30$.

Central Limit Theorem in Action





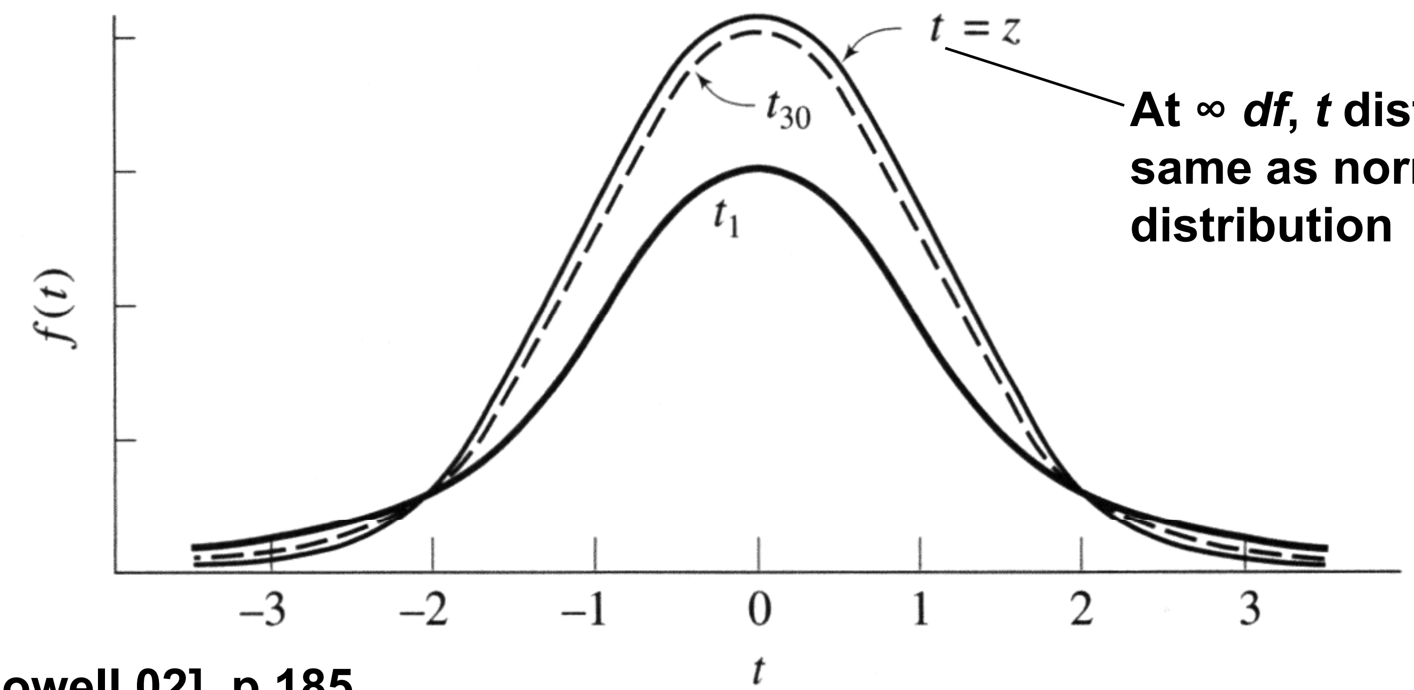
Response time data set *A*; *N* = 3436 data points. Data from [Living et al. 03].

Plotting 100 means drawn from *A* at random without replacement, where *n* is number of samples used to calculate mean.

- **This demonstrates:**
  - **As number of samples increases, distribution of means approaches normal distribution;**
  - **Regardless of how "non-normal" the source distribution is!**

The *t* Distribution

- In practice, when H_0: $\mu_c - \mu_d = 0$
  (two means come from same population),
  we calculate $\alpha = p(X \mid H_0)$ from *t* distribution, not *Z* distribution
- Why? *Z* requires the population parameter σ^2, but σ^2 almost never known. We estimate σ^2 with s^2, but s^2 biased to underestimate σ^2. Thus, *t* more spread out than *Z* distribution.
- *t* distribution parametric: parameter is *df* (degrees of freedom)



At ∞ *df*, *t* distribution same as normal distribution

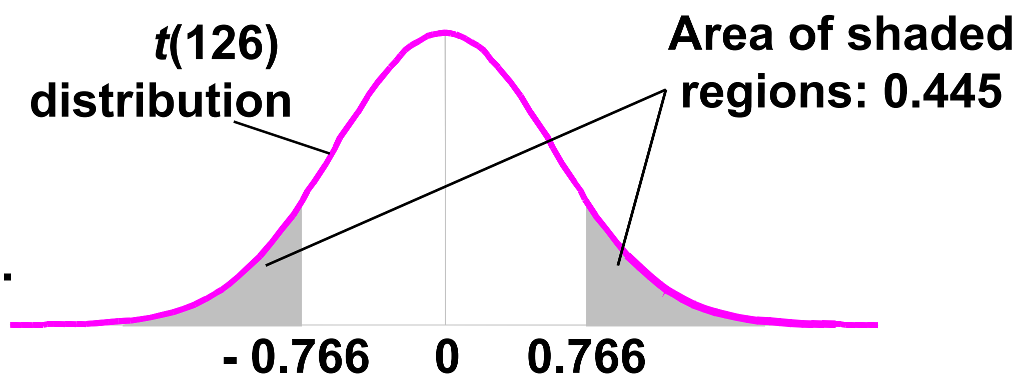
From [Howell 02], p 185

t-Test Example

- Null hypothesis H_0: $\mu_s - \mu_m = 0$
  - Subjects same speed whether stereo or mono viewing.
- Ran an experiment and collected samples:
  - 32 subjects, collected 128 samples
  - n_s = 64, X_s = 36.431 sec, s_s = 15.954 sec
  - n_m = 64, X_m = 34.449 sec, s_m = 13.175 sec

$$t(126) = \frac{f\left(\overline{X}\right)}{f\left(s^2, N\right)} = \frac{\overline{X}_s - \overline{X}_m}{\sqrt{s_p^2\left(\frac{1}{n_s} + \frac{1}{n_m}\right)}} = 0.766, \quad s_p^2 = \frac{(n_s - 1)s_s^2 + (n_m - 1)s_m^2}{n_s + n_m - 2}$$

- Look up $t(126) = 0.766$ in a *t*-distribution table: 0.445
- Thus, $\alpha = p($ 1.983 sec $| H_0) =$ 0.445, and we do not reject H_0.



Calculation described by [Howell 02], p 202

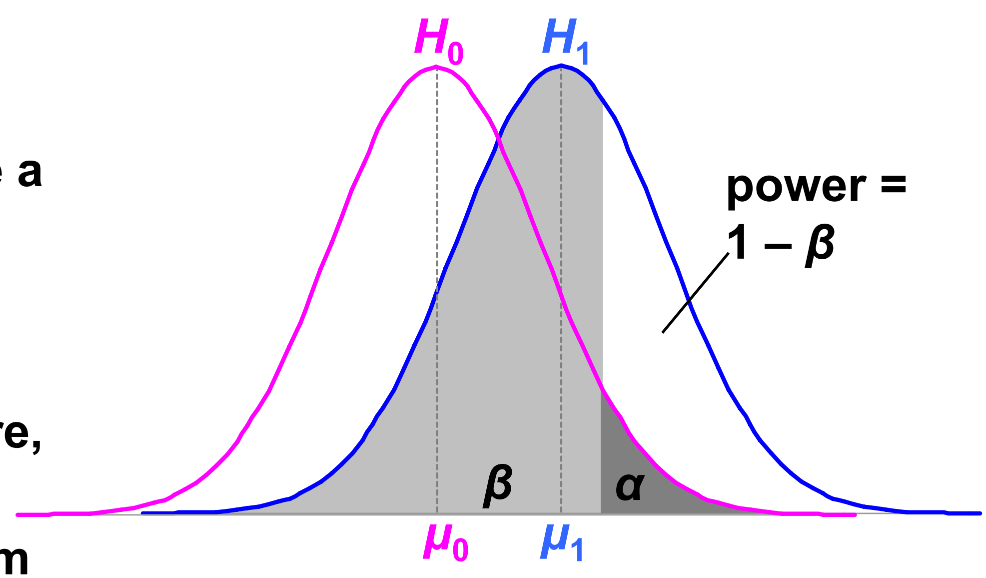
Power

- Empiricism
- Experimental Validity
- Experimental Design
- Gathering Data
- Describing Data
  - Graphing Data
  - Descriptive Statistics
- Inferential Statistics
  - Hypothesis Testing
  - Hypothesis Testing Means
  - *Power*
  - Analysis of Variance and Factorial Experiments

Interpreting α, β, and Power

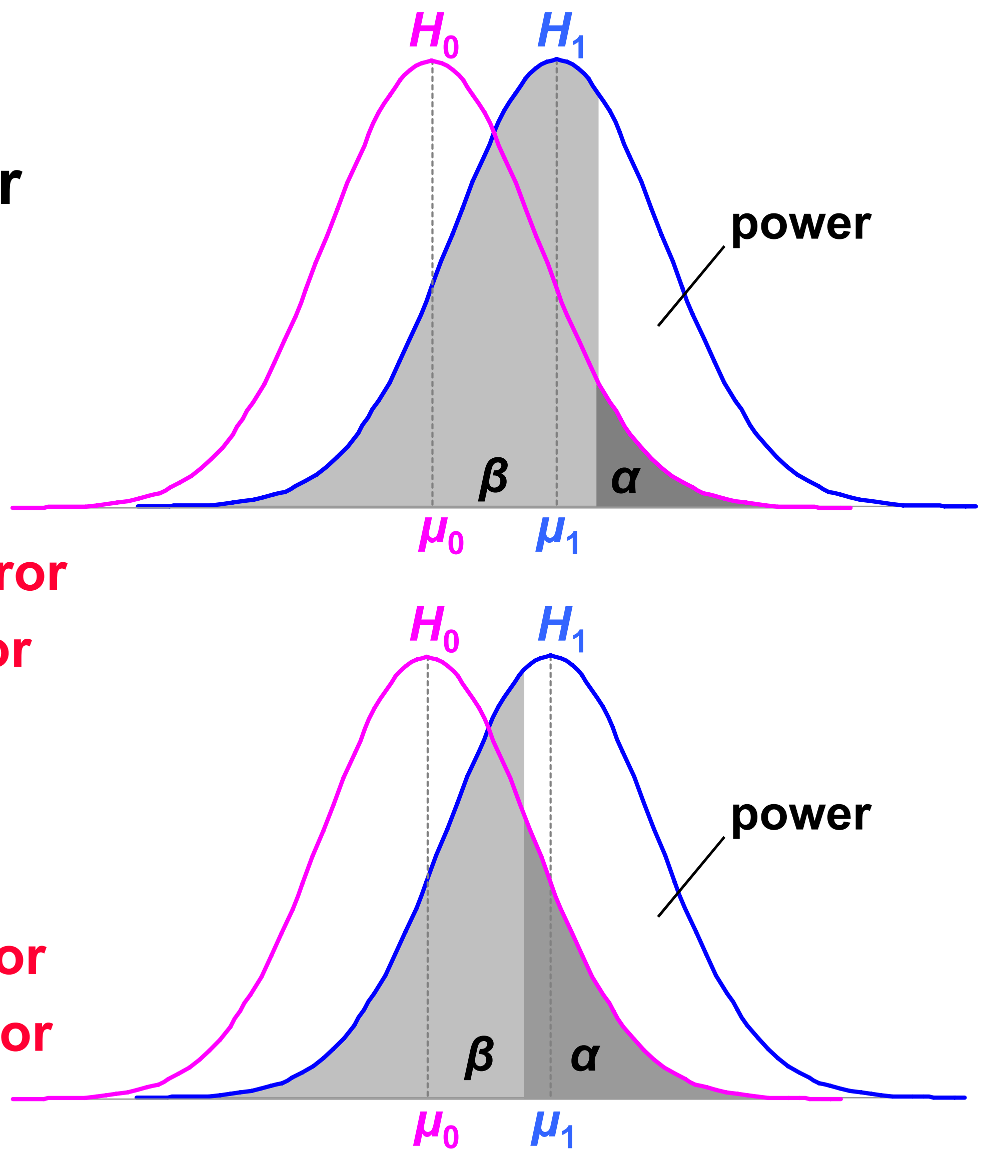
		Decision	
		Reject H_0	Don't reject H_0
True state of the world	H_0 false	a result! $p = 1 - \beta$ = power	type II error $p = \beta$
	H_0 true	type I error $p = \alpha$	wasted time $p = 1 - \alpha$

- If H_0 is true:
  - α is probability we make a type I error: we think we have a result, but we are wrong
- If H_1 is true:
  - β is probability we make a type II error: a result was there, but we missed it
  - Power is a more common term than β



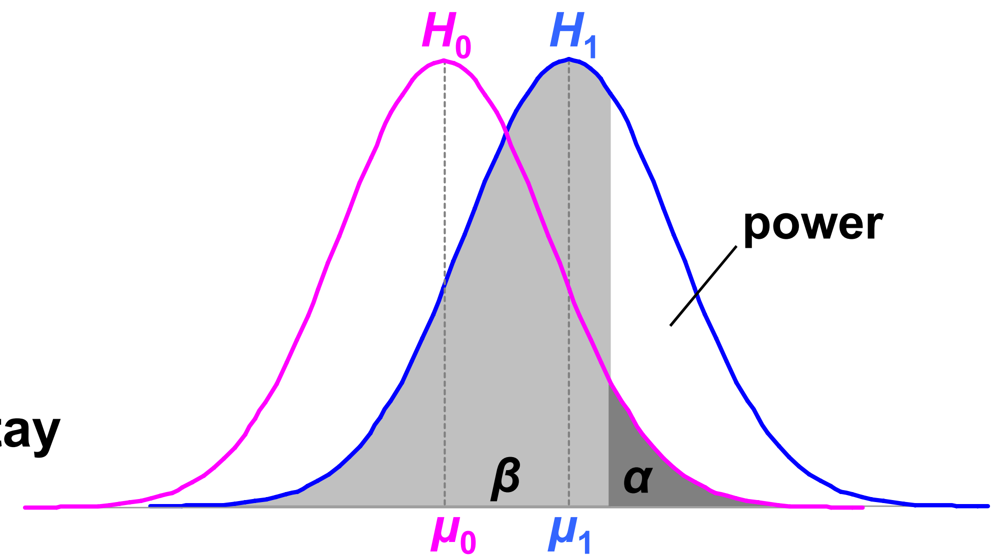
Increasing Power by Increasing α

- **Illustrates α / power tradeoff**
- **Increasing α:**
  - Increases power
  - Decreases type II error
  - Increases type I error
- **Decreasing α:**
  - Decreases power
  - Increases type II error
  - Decreases type I error

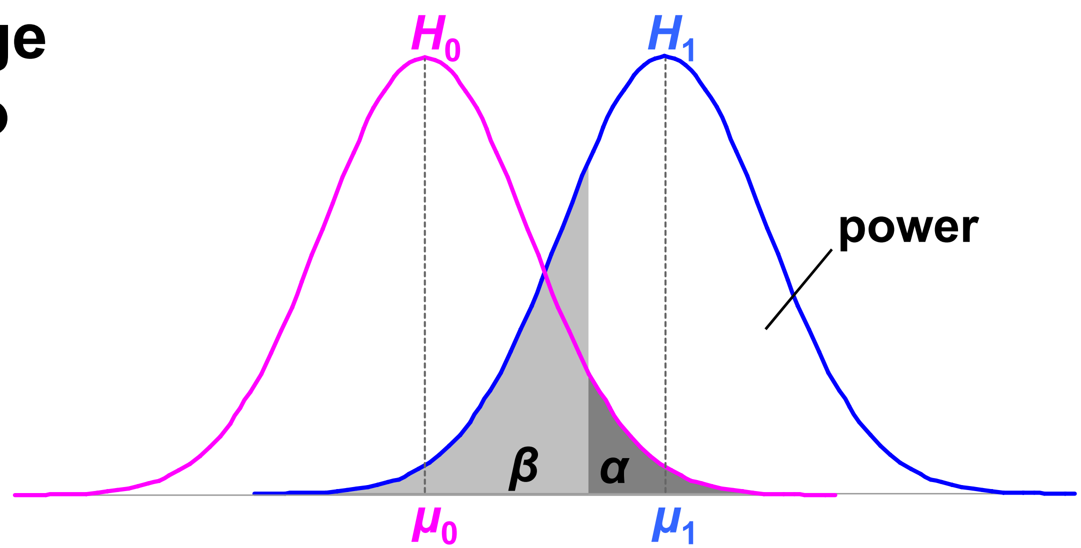


Increasing Power by Measuring a Bigger Effect

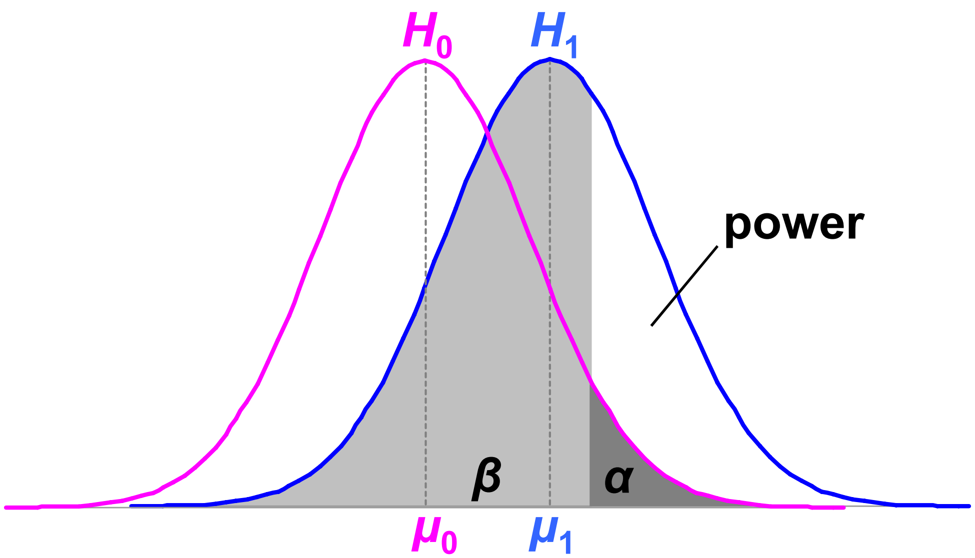
- **If the effect size is large:**
  - Power increases
  - Type II error decreases
  - α and type I error stay the same



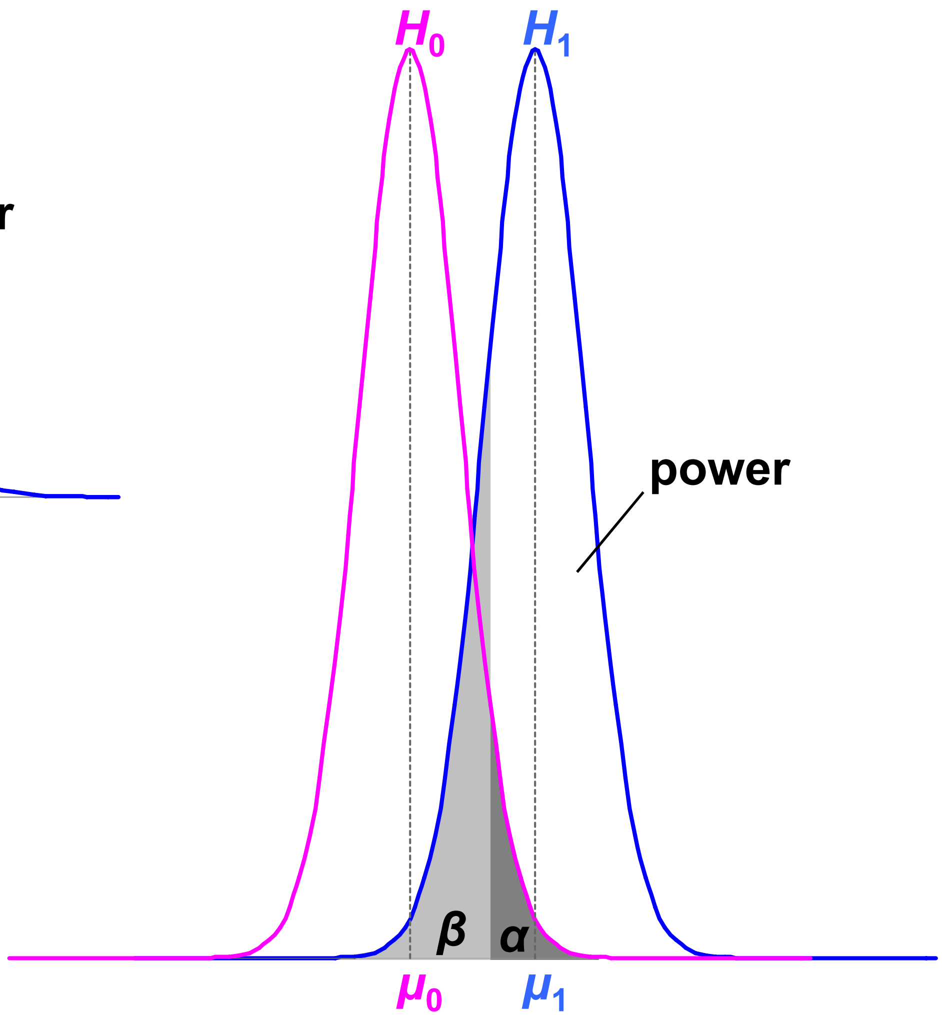
- **Unsurprisingly, large effects are easier to detect than small effects**



Increasing Power by Collecting More Data



- **Increasing sample size (*N*):**
  - **Decreases variance**
  - **Increases power**
  - **Decreases type II error**
  - ***α* and type I error stay the same**
- **There are techniques that give the value of *N* required for a certain power level.**



- **Here, effect size remains the same, but variance drops by half.**

Using Power

- Need α, effect size, and sample size for power:

$$\text{power} = f(\ \alpha,\ |\mu_0 - \mu_1|,\ N\ )$$

- Problem for Visualization:
  - Effect size $|\mu_0 - \mu_1|$ hard to know in our field
    - Population parameters estimated from prior studies
    - But our field is so new, not many prior studies
  - Can find effect sizes in more mature fields

- Post-hoc power analysis:

$$\text{effect size} = |X_0 - X_1|$$

  - Estimate from sample statistics
  - But this makes statisticians grumble (e.g. [Howell 02] [Cohen 88])

Other Uses for Power

1. Number samples needed for certain power level:

   $$N = f(\text{ power}, \alpha, |\mu_0 - \mu_1| \text{ or } |X_0 - X_1|\ )$$

   – Number extra samples needed for more powerful result
   – Gives "rational basis" for deciding *N* [Cohen 88]

2. Effect size that will be detectable:

   $$|\mu_0 - \mu_1| = f(\ N, \text{power}, \alpha\ )$$

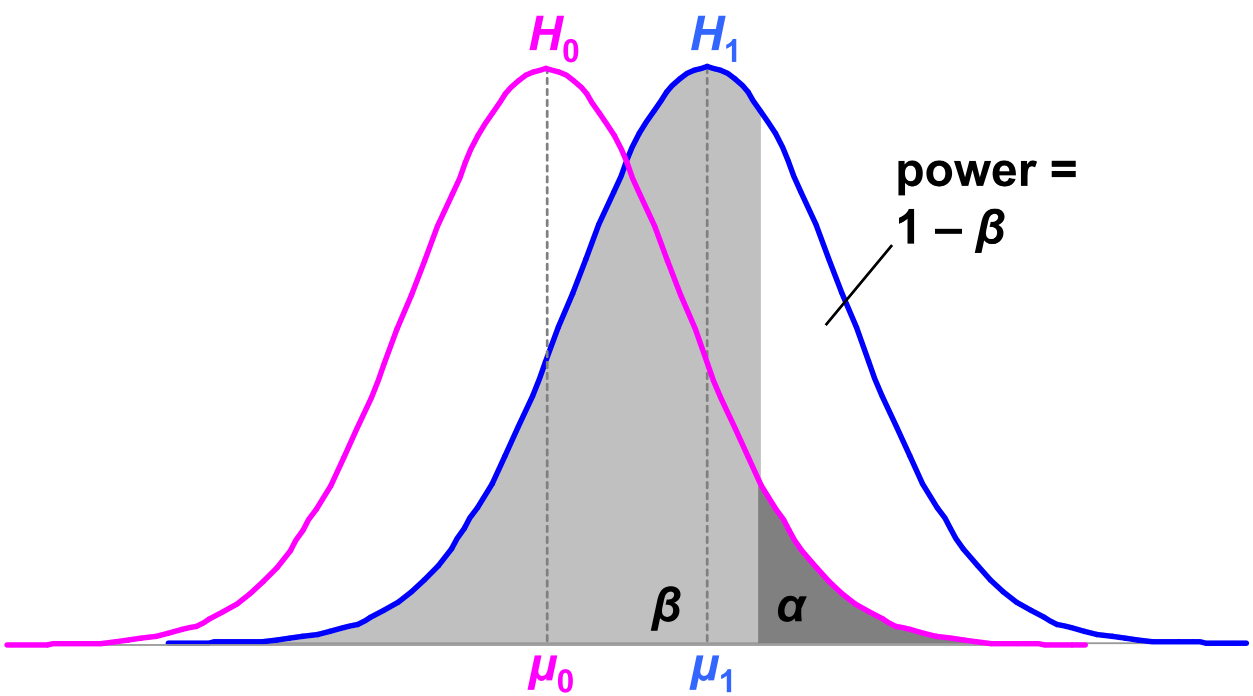
3. Significance level needed:

   $$\alpha = f(\ |\mu_0 - \mu_1| \text{ or } |X_0 - X_1|, N, \text{power}\ )$$

(1) is the most common power usage

Arguing the Null Hypothesis

- Cannot directly argue H_0: $\mu_s - \mu_m = 0$. But we can argue that $|\mu_0 - \mu_1| < d$.
  - Thus, we have bound our effect size by *d*.
  - If *d* is *small*, effectively argued null hypothesis.



From [Cohen 88], p 16

Example of Arguing H_0

- **We know GP is effective depth cue, but can we get close with other graphical cues?**

ground plane	drawing style	opacity	intensity	mean error*
on	all levels	both levels	both levels	0.144
off	wire+fill	decreasing	decreasing	0.111

$^*F(1,1870) = 1.002$, $p = .317$

- **Our effect size is $d = .087$ standard deviations**

- **Where can our experiment bound d?**

  $d(N = 265, \text{power} = .95, \alpha = .05) = .31$ standard deviations

- **This bound is significant at $\alpha = .05$, $\beta = .05$, using same logic as hypothesis testing.**
  But how meaningful is $d \geq .31$? Other significant d's:

  .37, .12, .093, .19

- **Not very meaningful. If we ran an experiment to bound $d \geq .1$, how much data would we need?**

  $N(\text{power} = .95, \alpha = .05, d = .1) = 2600$

- **Original study collected $N = 3456$, so $N = 2600$ reasonable**

Data from [Living et al. 03]

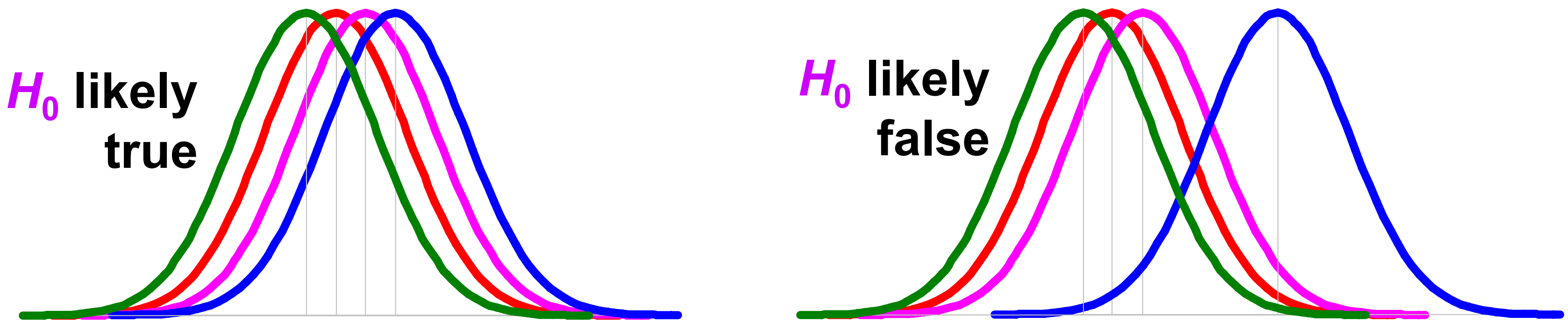
Analysis of Variance and Factorial Experiments

- Empiricism
- Experimental Validity
- Experimental Design
- Gathering Data
- Describing Data
  - Graphing Data
  - Descriptive Statistics
- Inferential Statistics
  - Hypothesis Testing
  - Hypothesis Testing Means
  - Power
  - *Analysis of Variance and Factorial Experiments*

ANOVA: Analysis of Variance

- *t*-test used for comparing two means
  - (2 x 1 designs)
- ANOVA used for factorial designs
  - Comparing multiple levels (*n* x 1 designs)
  - Comparing multiple independent variables (*n* x *m*, *n* x *m* x *p*), etc.
  - Can also compare two levels (2 x 1 designs); ANOVA can be considered a generalization of a *t*-Test
- No limit to experimental design size or complexity
- Most widely used statistical test in psychological research
- ANOVA based on the *F* distribution; also called an *F*-Test

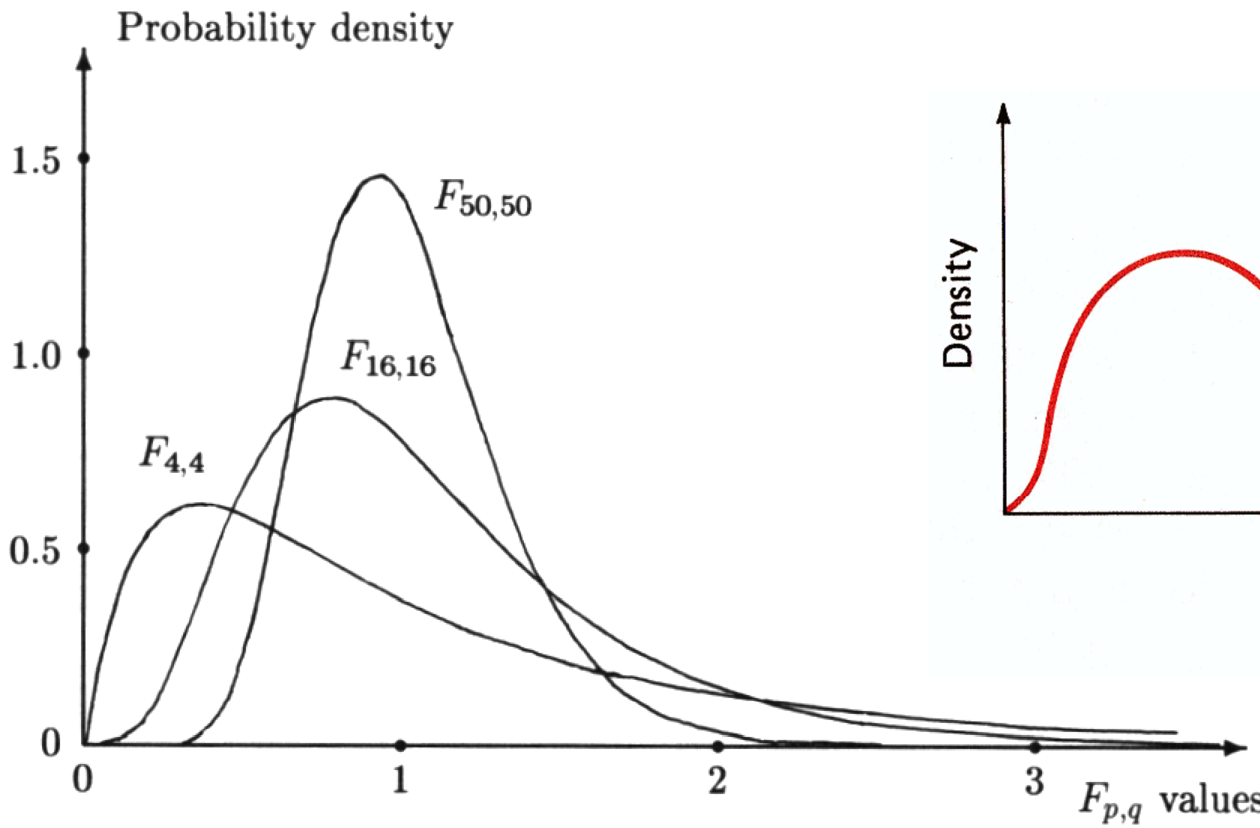
How ANOVA Works

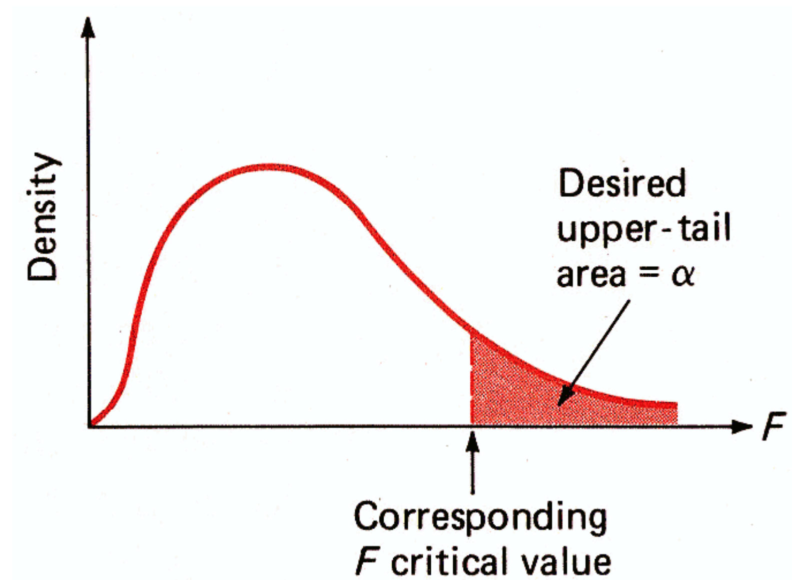


- Null hypothesis H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; H_1: at least one mean differs
- Estimate variance between each group: $MS_{between}$
  - Based on the difference between group means
  - If H_0 is true, accurate estimation
  - If H_0 is false, biased estimation: overestimates variance
- Estimate variance within each group: MS_{within}
  - Treats each group separately
  - Accurate estimation whether H_0 is true or false
- Calculate F critical value from ratio: $F = MS_{between} / MS_{within}$
  - If $F \approx 1$, then accept H_0
  - If $F >> 1$, then reject H_0

ANOVA Uses The *F* Distribution

- **Calculate $\alpha = p(X \mid H_0)$ by looking up *F* critical value in *F*-distribution table**
- ***F*-distribution parametric: *F* (numerator *df*, denominator *df*)**
- **α is area to right of *F* critical value (one-tailed test)**
- ***F* and *t* are distributions are related: $F(1, q) = t(q)^2$**

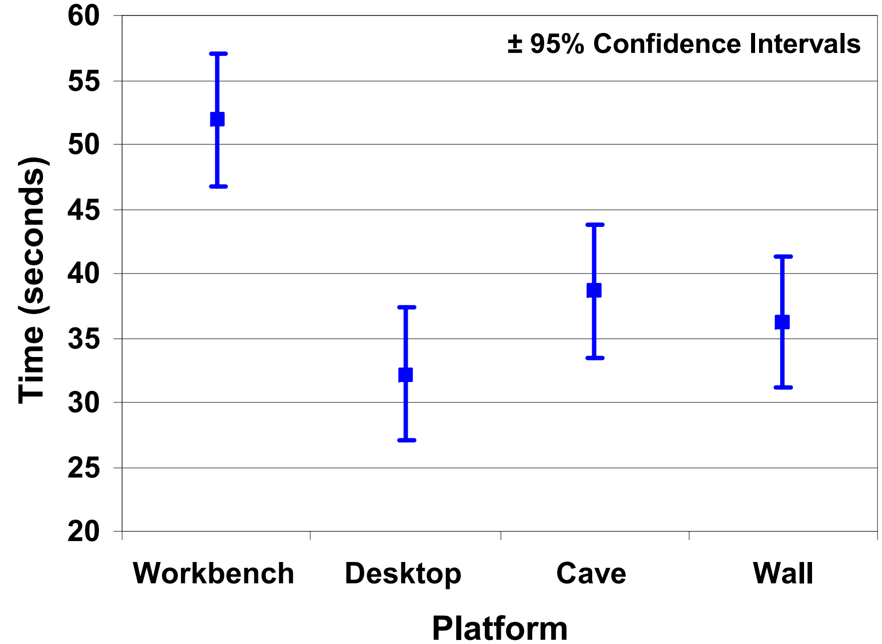




From [Saville Wood 91], p 52, and [Devore Peck 86], p 563

ANOVA Example

- Hypothesis H_1:
  - Platform (Workbench, Desktop, Cave, or Wall) will affect user navigation time in a virtual environment.
- Null hypothesis H_0: $\mu_b = \mu_d = \mu_c = \mu_w$.
  - Platform will have no effect on user navigation time.
- Ran 32 subjects, each subject used each platform, collected 128 data points.



Source	SS	*df*	MS	*F*	*p*
Between (platform)	1205.8876	3	401.9625	3.100*	0.031
Within (P x S)	12059.0950	93	129.6677		

*$p < .05$

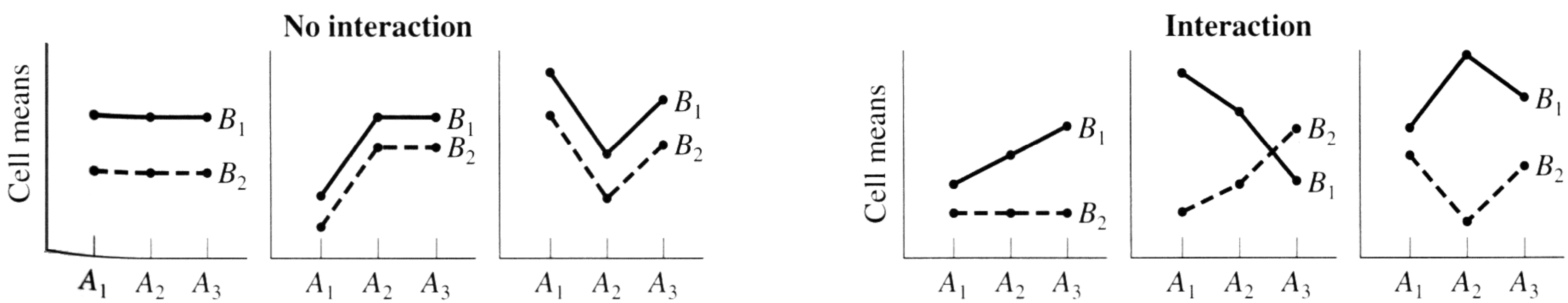
- Reporting in a paper: $F(3, 93) = 3.1$, $p < .05$

Data from [Swan et al. 03], calculations shown in [Howell 02], p 471

Main Effects and Interactions

- **Main Effect**
  - The effect of a single independent variable
  - In previous example, a *main effect* of platform on user navigation time: users were slower on the Workbench, relative to other platforms

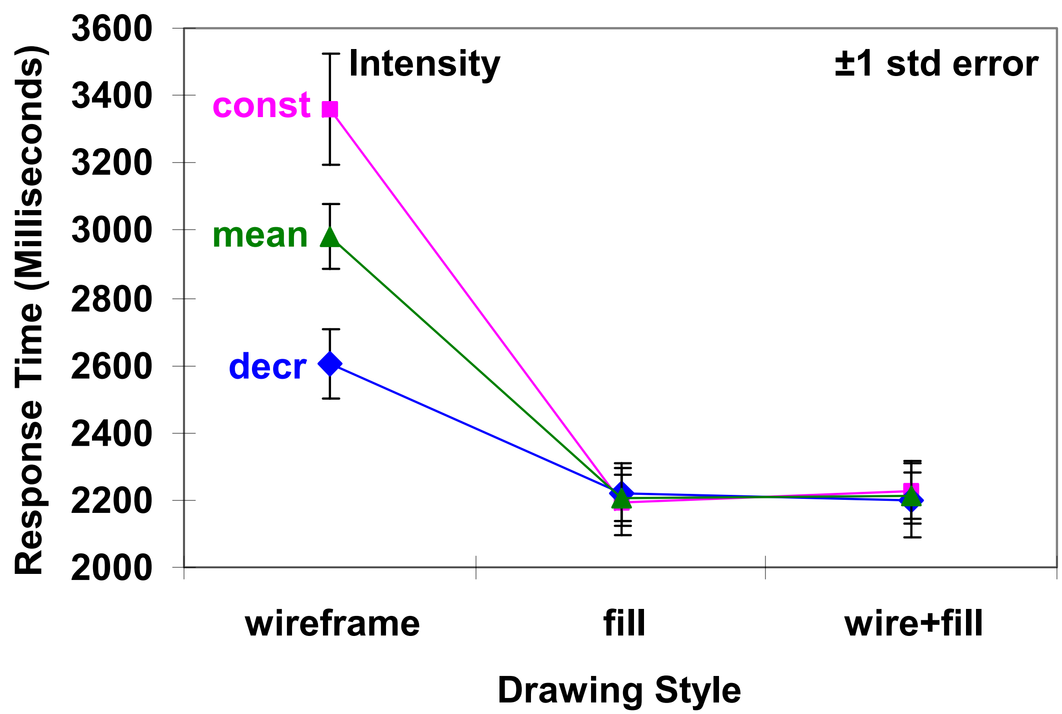
- **Interaction**
  - Two or more variables interact
  - Often, a 2-way interaction can describe main effects



From [Howell 02], p 431

Example of an Interaction

- **Main effect of drawing style:**
  - $F(2,14) = 8.84$, $p < .01$
  - Subjects slower with wireframe style
- **Main effect of intensity:**
  - $F(1,7) = 13.16$, $p < .01$
  - Subjects faster with decreasing intensity
- **Interaction between drawing style and intensity:**
  - $F(2,14) = 9.38$, $p < .01$
  - The effect of decreasing intensity occurs only for the wireframe drawing style; for fill and wire+fill, intensity had no effect
  - This completely describes the main effects discussed above



Data from [Living et al. 03]

Outline

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  - Graphing Data
  - Descriptive Statistics
- Inferential Statistics
  - Hypothesis Testing
  - Hypothesis Testing Means
  - Power
  - Analysis of Variance and Factorial Experiments

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